Corrections to Renormalisation Group Papers

July 1, 2020

Abstract

This is a list of corrections to various papers based on the renormalisation group, by Bauerschmidt, Brydges, Slade, Tomberg, Wallace.

R. Bauerschmidt, D.C. Brydges, and G. Slade. Scaling limits and critical behaviour of the 4-dimensional n-component $|\varphi|^4$ spin model. *J. Stat. Phys*, 157:692–742, (2014).

1. In (1.19), $\frac{\partial}{\partial m^2}p(0,m^2)$ should instead be the second derivative $\frac{\partial^2}{\partial (m^2)^2}p(0,m^2)$. The rest of the equation is correct after this change.

2. In (1.21), the right-hand side is missing a factor n. In the equality below (1.21), there is a factor $\frac{1}{2}n$ missing in front of $C_0(0)$. The factor $\frac{1}{2}$ got dropped from (4.50) in the first equality of (4.56) and the omission continues through the rest of the proof of Theorem 1.2(ii).

3. In the second and third paragraphs below Theorem 1.3: one reference to Theorem 1.3(ii) should be to (i), and two references to Theorem 1.3(iii) should be (ii).

4. There are several inconsequential[1] errors in (3.19)–(3.28) and we list their correct forms here with corrections in red:

$$\beta = (8+n)\delta[w^{(2)}], \qquad \theta = (2+n)\delta[(w^3)^{(**)}], \tag{1}$$
$$\xi' = 2(2+n)\big(\delta[w^{(3)}] - 3C_{0,0}w^{(2)}\big) + \gamma\beta\eta', \quad \pi' = (2+n)\delta[(-\Delta w w)^{(1)}], \tag{2}$$
$$\sigma = \tfrac{1}{2}(2+n)\delta[(\Delta w w)^{(**)}], \qquad \zeta = \tfrac{1}{2}(2+n)\delta[((\nabla w)^2)^{(**)}]. \tag{3}$$

$$g_{\rm pt} = g - \beta g^2 - 4g\delta[\nu w^{(1)}], \tag{4}$$
$$\nu_{\rm pt} = \nu + \eta'\big(g + 4g\nu w^{(1)}\big) - \gamma\beta g\nu - \xi' g^2 - \pi' g(y+z) - \delta[\nu^2 w^{(1)}] \tag{5}$$
$$y_{\rm pt} = y + \sigma z g - \zeta y g - \tfrac{1}{2}(n+2)g\delta[\nu(w^{(2)})^{(**)}] - y\delta[\nu w^{(1)}] \tag{6}$$
$$z_{\rm pt} = z + \theta g^2 + \tfrac{1}{2}\delta[\nu^2 w^{(**)}] - 2(y+z)\delta[\nu w^{(1)}] - (y_{\rm pt} - y). \tag{7}$$

[1] A method to estimate almost all of these quantities without knowing their explicit formulas is developed in Section 3 of R. Bauerschmidt, M. Lohmann, G. Slade, Three-dimensional tricritical spins and polymers, *J. Math. Phys.*, **61**:033302, (2020).

The κ coefficients defined by

$$\delta u_{\rm pt} = \kappa_g g + \kappa'_\nu \nu + \kappa_z (y+z) - \kappa_{gg} g^2 - \kappa'_{\nu\nu}\nu^2 - \kappa'_{g\nu} g\nu \\ - \kappa_{gz} g(y+z) - \kappa_{zz}(y+z)^2 - \kappa'_{\nu z}\nu(y+z), \tag{8}$$

are given by

$$\kappa_g = \frac{1}{4}n(n+2)C^2, \quad \kappa'_\nu = \tfrac{1}{2}nC, \quad \kappa_z = \tfrac{1}{2}n(-\Delta C). \tag{9}$$

$$\kappa_{gg} = \frac{1}{4}n(n+2)\Big(\delta[w^{(4)}] - 4Cw^{(3)} + 2(-\Delta C)(w^{(3)})^{(**)} - 6C^2 w^{(2)} + (n+2)C^2\delta[w^{(2)}]\Big), \tag{10}$$

$$\kappa'_{\nu\nu} = \frac{1}{4}n\Big(\delta[w^{(2)}] - 2Cw^{(1)} + (-\Delta C)w^{(**)}\Big), \tag{11}$$

$$\kappa'_{g\nu} = n(n+2)\Big(\tfrac{1}{2}C\delta[w^{(2)}] - C^2 w^{(1)}\Big), \tag{12}$$

$$\kappa_{gz} = \tfrac{1}{2}n(n+2)C\delta[(-\Delta w w)^{(1)}], \tag{13}$$

$$\kappa_{zz} = \frac{1}{4}n\delta[(\Delta w)^{(2)}], \tag{14}$$

$$\kappa'_{\nu z} = -n(-\Delta C)w^{(1)} + \tfrac{1}{2}n\delta[(-\Delta w w)^{(1)}]. \tag{15}$$

In (3.46) there are several inconsequential errors. The correct equation is:

$$\delta\bar{u}_+ = \kappa_g \bar{g} + \kappa_\mu \bar{\mu} - \kappa_z \bar{z} - \kappa_{gg}\bar{g}^2 - \kappa_{\mu\mu}\bar{\mu}^2 - \kappa_{g\mu}\bar{g}\bar{\mu} - \kappa_{gz}\bar{g}\bar{z} - \kappa_{zz}\bar{z}^2 - \kappa_{z\nu}\bar{z}\bar{\mu}.$$

5. Above (4.45): the reference to (4.7) should be (4.5).

6. Above (4.46): the reference to (4.7) should be (4.6).

7. In (4.49): all j on both right-hand sides should be N.

8. In the paragraph below (4.53): I should instead be a subset of $(\nu_c, \nu_c + \delta)$, and in the third-from-last sentence: "0 would also have to be a limit point" should instead be "ν_c would also have to be a limit point".

9. In (4.64), the L^{-Nd} in the middle and right-hand side should instead be $L^{-Nd/2}$ (subsequent calculations are correct after this change).

10. For (5.3) ((A.3) in arXiv version): replace Δ in the third term by $-\Delta$. Equations (5.4)–(5.6) are then correct.

D.C. Brydges and G. Slade. A renormalisation group method. I. Gaussian integration and normed algebras. *J. Stat. Phys*, 159:421–460, (2015).

1. In (3.34): $=$ should be $\leq$.

R. Bauerschmidt, D.C. Brydges, and G. Slade. A renormalisation group method. III. Perturbative analysis. *J. Stat. Phys*, 159:492–529, (2015).

1. First line of second paragraph of Section 2: $\mathbb{Z}^d/(L^N\mathbb{Z})$ should be $\mathbb{Z}^d/(L^N\mathbb{Z}^d)$.

2. Above (3.13) and above (6.7) there are references to Section 5.2 for the definition of Euclidean invariance, but it is not defined in Section 5.2. The definition is given above (3.13) and there is no need for further reference.

3. In (3.16): on the right-hand side $\partial/\partial\phi_v$ should be $\partial/\partial\bar{\phi}_v$. (This is correct in the arXiv version but the bar on $\bar{\phi}_v$ is absent in the published version.)

4. Equations (3.27)–(3.33) contain errors. The correct equations are obtained by setting $n = 0$ in (1)–(7).

5. In (5.23): the formula is correct in the arXiv version for polynomials V', V'' which are *even* in the fermions, but the bars are missing on the ϕ derivatives in the published version. The correct equation for polynomials which are even in the fermions is:

$$V' \overleftrightarrow{\mathcal{L}}_w V'' = \sum_{u,v\in\Lambda} w_{uv} \left(\frac{\partial V'}{\partial \phi_u} \frac{\partial V''}{\partial \bar{\phi}_v} + \frac{\partial V'}{\partial \phi_v} \frac{\partial V''}{\partial \bar{\phi}_u} + \frac{\partial V'}{\partial \psi_u} \frac{\partial V''}{\partial \bar{\psi}_v} + \frac{\partial V'}{\partial \psi_v} \frac{\partial V''}{\partial \bar{\psi}_u} \right).$$

   If V', V'' can be odd in the fermions then the signs of the fermionic terms require more care. In our applications, V', V'' are even in the fermions.

6. There are sign errors in (5.30) and (5.32) and these equations should read:

$$\mathrm{Loc}_x \left[\sum_{y\in\Lambda} q(x-y)(\tau_{xy} + \tau_{yx}) \right] = 2q^{(1)}\tau_x - q^{(**)}\tau_{\Delta,x}$$

   and, because the equation in the line above (5.32) should be $\Delta x_1^2 = +2$,

$$\sum_{x\in\Lambda} (\Delta q)_x x_1^2 = +2 \sum_{x\in\Lambda} q_x = +2q^{(1)}.$$

7. Three lines below (6.16): typo in “right-continuous”.

D.C. Brydges and G. Slade. A renormalisation group method. IV. Stability analysis. *J. Stat. Phys*, 159:530–588, (2015).

1. Three lines below (1.43): delete “we” after “whereas”.

2. In the statement of Proposition 2.8, $\mathcal{D}$ should be $\bar{\mathcal{D}}$ (it is correct with $\bar{\mathcal{D}}$ in Proposition 7.2).

3. Typo above Lemma 7.1: $C_{\delta L}$ should be $C_{\delta V}$.

4. Two lines below (7.15): in the norms of C, the $\hat{\ell}$ on the left-hand side should be ℓ, and the ℓ on the right-hand side should be $\hat{\ell}$.

5. In (A.20): $f \in V$ should be $f \in Q$.

D.C. Brydges and G. Slade. A renormalisation group method. V. A single renormalisation group step. *J. Stat. Phys*, 159:589–667, (2015).

1. (1.1) should instead read
$$\{x = (x_1, \ldots, x_d) \in \Lambda : 0 \leq x_i < L^d \text{ for } i = 1, \ldots, d\}$$
2. In Proposition 1.5: $\mathbb{E}_1$ on the left-hand side of (1.23) should be replaced by $\mathbb{E}_1\theta$. In addition, I_1 should be replaced in the statement and proof by $\tilde{I}_1$, to denote that it has been assumed to factorise over blocks at scale-0 and not at scale-1 as specified in (1.23). The additional manoeuvre required to adjust to a scale-1 I_1 is discussed in Section 6.1; however this is not the point of Proposition 1.5 which is intended only to be illustrative. The role of scale-0 factorisation is discussed more clearly in Proposition 12.4.2 of Bauerschmidt, Brydges and Slade, "Introduction to a Renormalisation Group Method," Springer Lecture Notes in Mathematics Vol. 2242, (2019).
3. Four lines below (2.7): it should read $\|Q(B)\|_{T_0(\ell)} \leq O(r_Q)$ (and not $O(r_0)$ in the upper bound).
4. (3.29): in line above, delete "of".
5. (4.23): should read $K^{(1)}(B) = K(B) - I^B J(B, B)$. (The I^B was missing, and the sum over U should not be present.)
6. (5.35): right-hand side should be $r^{(2)}\bar{\epsilon}^{1+f_j(a^{(2)},X)}$.
7. (5.39): in the second line, the term -2^d in the exponent should be $-a^{(2)}2^d$, and in the last line, f_j should be f_{j+1}.
8. In (5.45), 3 times in (5.47), and in (5.48): $\mathcal{X}(X)$ should be $\mathcal{X}(U)$.
9. Twice in (5.47), and once in (5.48): $|X|$ should be $|U|$.
10. In (5.47): the last exponent should be $\sum_i (1 + f_j(a^{(2)}, X_{K,i})$.
11. Below (5.49), the definition of b should have $\bar{\epsilon}^\delta$ instead of $\bar{\epsilon}$.
12. Proposition B.2: the domain of the map should be $\tilde{\mathbb{I}}_+(\tilde{m}^2)$ rather than $\mathcal{X}$. Also, the reference to [4, (1.15)] should be [4, (1.35)].
13. (D.14): should read $K_{\text{out}}(B) = M(B)$ (with no sum over U). In fact, when $W = B$, we have $X = \varnothing$ and $\mathcal{U}(X) = \varnothing$, so $X^\square = \varnothing$, $U_M = B$ and $\mathcal{Y} = \{(B, \varnothing, B)\}$ in (D.13).

R. Bauerschmidt, D.C. Brydges, and G. Slade. Structural stability of a dynamical system near a non-hyperbolic fixed point. *Ann. Henri Poincaré*, 16:1033–1065, (2015).

The authors are grateful to Satoshi Handa for pointing out the need for these corrections (they do not have larger impact):

1. Above (2.9): replace $\sum_{n=1}^{\infty} \Omega^{-n}$ by $\|\beta\|_\infty \sum_{n=1}^{\infty} \Omega^{-n}$.

2. In (2.31): the right-hand side should instead be $c_j \times O(\chi_l \bar{g}_l)$.

3. The sentence containing (2.32) should be replaced by the following: For $j \geq j_\Omega$, we use $1/(1-x) \leq e^{2|x|}$ for small $|x|$ to obtain

$$\prod_{k=j}^{l} (1 - \zeta_k \bar{g}_k)^{-1} \leq \exp\left[2 \sum_{k=j}^{l} |\zeta_k| \bar{g}_k\right] \leq \exp\left[C \bar{g}_j \sum_{k=j_\Omega}^{\infty} \chi_k\right] \leq O(1). \tag{16}$$

4. In (2.51): the product should instead have limits $\prod_{k=l+1}^{j-1}$.

5. In (2.57): A factor σ_l is missing on the right-hand side, it should multiply $\sum_{i=j}^{l} (\lambda - \tau)^{-1} \tau'$.

R. Bauerschmidt, D.C. Brydges, and G. Slade. Logarithmic correction for the susceptibility of the 4-dimensional weakly self-avoiding walk: a renormalisation group analysis. *Commun. Math. Phys.*, 337:817–877, (2015).

1. In (8.6): on the left-hand side K_j should be K_{j+1}.

2. Below (8.63): four factors $(1-\gamma)^{-1}$ should instead all be $(1-\gamma)$ (this has no effect).

G. Slade and A. Tomberg. Critical correlation functions for the 4-dimensional weakly self-avoiding walk and n-component $|\varphi|^4$ model. *Commun. Math. Phys.*, 342:675–737, (2016).

1. In the last line of (4.31): $O(\chi_j)$ should be $O(L^{2j} \chi_j)$.

2. In (4.32): the left-hand side should be $\delta_j' - \delta_j$.

3. In the last line of the paragraph containing (4.32): δ_j' should be $\delta_j' - \delta_j$.

4. In (4.38): a subscript j should be i.

5. In (4.41): it is claimed that $\delta_i' = O(\chi_i \bar{g}_i^2)$, but it has only been proved that $\delta_i' = O(\chi_i \bar{g}_i)$. Thus (4.41) should be left as $E_i = (4\gamma - p)\delta_i' + O(\chi_i \bar{g}_i^2)$. To complete the proof, we use the fact that $\sum_i \delta_i'$ is a telescoping sum. It may not be absolutely convergent, but $\sum_i (\delta_i')^2$ is convergent, and (conditional) convergence plus absolute convergence of the square guarantees the convergence of the infinite product defining α_i in (4.42). (The general fact about infinite products used here can be found in Theorem 9, p.224 of Knopp's "Theory and Application of Infinite Series" (1954), `https://archive.org/details/theoryandapplica031692mbp/page/n237`.)

R. Bauerschmidt, G. Slade, A. Tomberg, and B.C. Wallace. Finite-order correlation length for 4-dimensional weakly self-avoiding walk and $|\varphi|^4$ spins. *Ann. Henri Poincaré*, 18:375–402, (2017).

1. The first bound of (3.27) should be $\|R_+(B)\|_{T_{0,+}} \le M\tilde{\vartheta}_+\tilde{g}_+^3$, i.e., a bound on the T_0 norm rather than the $\mathcal{V}$ norm. The reason is as follows: [18, Theorem 5.1] and [13, Theorems 1.10–1.11] are stated in terms of the $\mathcal{V}$ norm but the proof of those theorems uses the T_0 norm; the two norms are equivalent in those references. The proof of (3.27) therefore also uses the T_0 norm, but now the two norms are not equivalent for the bulk parameters because of the new ℓ_j in (3.23) when $s > 0$. Thus, the norm in the first inequality of (3.27) should be the T_0 norm, which gives a *worse* bound for the bulk parameters in R_+. However those bulk parameters have already been analysed and controlled in [2,4], and the estimates proved in those references of course continue to hold for the bulk part of R_+. On the other hand, the estimates for the observable part of R_+ are *improved* with the new norm, and this is what is used to prove Proposition 2.1.

2. The symbol $\mathbb{1}$ is missing in many places in Sections 4 and 5 of the published version, in exponents which should contain $\mathbb{1}_{j\ge j_m}$.

R. Bauerschmidt, G. Slade, and B.C. Wallace. Four-dimensional weakly self-avoiding walk with contact self-attraction. *J. Stat. Phys*, 167:317–350, (2017).

1. The second equation of (4.32) should instead read:

$$R^+ = 2(\nabla^e|\phi_x|^2)\nabla^e(\psi_x\bar{\psi}_x) - 2\psi_{x+e}\bar{\psi}_{x+e}\psi_x\bar{\psi}_x.$$

2. (4.69): The factor $\mathfrak{h}_0^8$ should be removed from the right-hand side (it does not make sense for it to appear since the $\mathcal{W}_0$ norm does not have a parameter $\mathfrak{h}_0$). For the term involving the G_0 norm in (4.57), $\mathfrak{h}_0 = \ell_0$ is an L-dependent constant. For the term involving the $\tilde{G}_0$ norm in (4.57), $\mathfrak{h}_0 = h_0 = k_0\tilde{g}_0^{-1/4}$, and its eighth power is cancelled by the factor $\tilde{g}_0^{9/4}$ in (4.57).

   This correction does not affect the application of Proposition 4.6 in Section 5.4.

3. In (5.43): Replace $(m^2, \beta, 0, g_0^*)$ by $(m^2, g_0^*, 0)$.