ST TABLE

ALEX COHEN

Variables

- $|\mathcal{P}|$
- $|\mathcal{L}|$
- r_P = Number of lines through each point
- r_L = Number of points on each line
- $\mathcal{I}(P,L) = r_P|\mathcal{P}| = r_L|\mathcal{L}|$
- $|\pi_\theta(P)| = \frac{|\mathcal{P}|}{r_L}$

We consider the Szemeredi-Trotter bound, two Cauchy-Schwarz bounds, and one high-low bound.

$\mathcal{P}$	$\mathcal{L}$	r_P	r_L	$\mathcal{I}$	$\lvert\pi_\theta(P)\rvert$	ST	CS(Lines)	CS(Points)	High case
1	✓	✓				$\lvert\mathcal{P}\rvert \lesssim \frac{\lvert\mathcal{L}\rvert^2}{r_P^3}$	$\lvert\mathcal{P}\rvert \lesssim \frac{\lvert\mathcal{L}\rvert^2}{r_P^2}$		$\lvert\mathcal{P}\rvert \lesssim \delta^{-1}\frac{\lvert\mathcal{L}\rvert}{r_P^2}$
✓	1	✓				$\lvert\mathcal{L}\rvert \gtrsim \lvert\mathcal{P}\rvert^{1/2} r_P^{3/2}$	$\lvert\mathcal{L}\rvert \gtrsim \lvert\mathcal{P}\rvert^{1/2} r_P$	$\lvert\mathcal{L}\rvert \gtrsim r_P^2$	$\lvert\mathcal{L}\rvert \gtrsim \delta\lvert\mathcal{P}\rvert\, r_P^2$
✓	✓	1				$r_P \lesssim \lvert\mathcal{L}\rvert^{2/3}\lvert\mathcal{P}\rvert^{-1/3}$	$r_P \lesssim \lvert\mathcal{L}\rvert\lvert\mathcal{P}\rvert^{-1/2}$	$r_P \lesssim \lvert\mathcal{L}\rvert^{1/2}$	$r_P \lesssim \lvert\mathcal{L}\rvert^{1/2}\lvert\mathcal{P}\rvert^{-1/2}\delta^{-1/2}$
1	✓		✓			$\lvert\mathcal{P}\rvert \gtrsim \lvert\mathcal{L}\rvert^{1/2} r_L^{3/2}$	$\lvert\mathcal{P}\rvert \gtrsim r_L^2$	$\lvert\mathcal{P}\rvert \gtrsim \lvert\mathcal{L}\rvert^{1/2} r_L$	$\lvert\mathcal{P}\rvert \gtrsim \delta\lvert\mathcal{L}\rvert r_L^2$
✓	1		✓			$\lvert\mathcal{L}\rvert \lesssim \frac{\lvert\mathcal{P}\rvert^2}{r_L^3}$		$\lvert\mathcal{L}\rvert \lesssim \frac{\lvert\mathcal{P}\rvert^2}{r_L^2}$	$\lvert\mathcal{L}\rvert \lesssim \delta^{-1}\frac{\lvert\mathcal{L}\rvert}{r_L^2}$
✓	✓		1			$r_L \lesssim \lvert\mathcal{P}\rvert^{2/3}\lvert\mathcal{L}\rvert^{-1/3}$	$r_L \lesssim \lvert\mathcal{P}\rvert^{1/2}$	$r_L \lesssim \lvert\mathcal{P}\rvert\lvert\mathcal{L}\rvert^{-1/2}$	$r_L \lesssim \lvert\mathcal{P}\rvert^{1/2}\lvert\mathcal{L}\rvert^{-1/2}\delta^{-1/2}$
1		✓	✓			$\lvert\mathcal{P}\rvert \gtrsim r_P r_L^2$	$\lvert\mathcal{P}\rvert \gtrsim r_L^2$	$\lvert\mathcal{P}\rvert \gtrsim r_P r_L$	$r_P r_L \lesssim \delta^{-1}$
✓		1	✓			$r_P \lesssim \frac{\lvert\mathcal{P}\rvert}{r_L^2}$		$r_P \lesssim \frac{\lvert\mathcal{P}\rvert}{r_L}$	$r_P \lesssim \frac{\delta^{-1}}{r_L}$
✓		✓	1			$r_L \lesssim \left(\frac{\lvert\mathcal{P}\rvert}{r_P}\right)^{1/2}$	$r_L \lesssim \lvert\mathcal{P}\rvert^{1/2}$	$r_L \lesssim \frac{\lvert\mathcal{P}\rvert}{r_P}$	$r_L \lesssim \frac{\delta^{-1}}{r_P}$
	1	✓	✓			$\lvert\mathcal{L}\rvert \gtrsim r_L r_P^2$	$\lvert\mathcal{L}\rvert \gtrsim r_P r_L$	$\lvert\mathcal{L}\rvert \gtrsim r_P^2$	$r_P r_L \lesssim \delta^{-1}$
	✓	1	✓			$r_P \lesssim \left(\frac{\lvert\mathcal{L}\rvert}{r_L}\right)^{1/2}$	$r_P \lesssim \frac{\lvert\mathcal{L}\rvert}{r_L}$	$r_P \lesssim \lvert\mathcal{L}\rvert^{1/2}$	$r_P \lesssim \frac{\delta^{-1}}{r_L}$
	✓	✓	1			$r_L \lesssim \frac{\lvert\mathcal{L}\rvert}{r_P^2}$	$r_L \lesssim \frac{\lvert\mathcal{L}\rvert}{r_P}$		$r_L \lesssim \frac{\delta^{-1}}{r_P}$
✓	✓			1		$\mathcal{I} \lesssim \lvert\mathcal{P}\rvert^{2/3}\lvert\mathcal{L}\rvert^{2/3}$	$\mathcal{I} \lesssim \lvert\mathcal{L}\rvert\lvert\mathcal{P}\rvert^{1/2}$	$\mathcal{I} \lesssim \lvert\mathcal{P}\rvert\lvert\mathcal{L}\rvert^{1/2}$	$\mathcal{I} \lesssim \delta^{-1/2}\lvert\mathcal{P}\rvert^{1/2}\lvert\mathcal{L}\rvert^{1/2}$
1		✓			✓	$\lvert\mathcal{P}\rvert \lesssim \frac{\lvert\pi_\theta(P)\rvert^2}{r_P}$	$\lvert\mathcal{P}\rvert \lesssim \lvert\pi_\theta(P)\rvert^2$		$\lvert\mathcal{P}\rvert \lesssim \delta^{-1}\frac{\lvert\pi_\theta(P)\rvert}{r_P}$
✓		1			✓	$r_P \lesssim \frac{\lvert\pi_\theta(P)\rvert^2}{\lvert\mathcal{P}\rvert}$		$r_P \lesssim \lvert\pi_\theta(P)\rvert$	$r_P \lesssim \delta^{-1}\frac{\lvert\pi_\theta(P)\rvert}{\lvert\mathcal{P}\rvert}$
✓		✓			1	$\lvert\pi_\theta(P)\rvert \gtrsim \lvert\mathcal{P}\rvert^{1/2} r_P^{1/2}$	$\lvert\pi_\theta(P)\rvert \gtrsim \lvert\mathcal{P}\rvert^{1/2}$	$\lvert\pi_\theta(P)\rvert \gtrsim r_P$	$\lvert\pi_\theta(P)\rvert \gtrsim \delta\lvert\mathcal{P}\rvert r_P$

TABLE 1. Incidence bounds in different forms