SYMMETRIC CONSTRAINTS IN CLASSIFICATION PROBLEMS THAT PERMIT REPLACEMENT BY FUNCTIONAL CONSTRAINTS

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Symmetric universal constraints for algorithm classification are described that, in a sense, permit replacement by equivalent functional constraints. Symmetric and functional constraints are a part of a theory of universal and local constraints [1-4] being developed within the framework of an algebraic approach [5, 6] to the problem of synthesis of correct recognition algorithms. The terminology, results, and notation described earlier [7] are employed here.

Let a functional category Φ that is defined by a permissible functional signature φ be given. Rudakov [3] has described the construction of a group σ_φ such that category Φ is Γ-complete in the symmetric category Σ_φ defined by that group. The condition

$$\forall (i_1, j_1)\, S_{(i_1, j_1)} \subset S_{(i_2, j_2)} \Rightarrow |\lambda^{-1}(\lambda(i_1, j_1))| = 1 \tag{1}$$

must be satisfied for signature φ.

In condition (1), inclusion is strict, i.e., $S_{(i_1, j_1)} \neq S_{(i_2, j_2)}$.

The question arises as to when the inverse problem can be solved — namely, that of constructing for a given group σ a functional signature φ such that group σ coincides with σ_φ. In other words, for a given symmetric category Σ we can construct a functional category Φ such that Φ is a Γ-complete subcategory of Σ, where Σ includes any symmetric category in which category Φ is Γ-complete. Below we shall describe a signature for groups σ that permit such a construction. We shall examine only groups σ that do not have stationary elements. As was pointed out earlier [7], this does not reduce the generality of the examination.

Definition 1. Signature φ is called maximal for group σ if for any pair (i_1, j_1) of S set $S_{(i_1, j_1)}$ coincides with a block $w_{(i_1, j_1)}$, constructed for group σ.

We shall describe the construction of functional signature φ_σ.

1. We number arbitrarily the classes of functional similarity in S for group σ. For each pair $S_{(i_1, j_1)}$ contained in class γ^l with number l, we let $\lambda_{(i_1, j_1)}$ be equal to l.

2. In each class γ^l we select an arbitrary pair (i_1, j_1), which we call the lead pair. We define $S_{(i_l, j_l)}$ as follows: $S_{(i_l, j_l)}$ coincides element by element with block $w_{(i_l, j_l)}$; order in $S_{(i_l, j_l)}$ is introduced arbitrarily.

3. For all pairs (i, j) of class γ^l we let $S_{(i,j)}$ be equal to $g(S_{(i_l, j_l)})$, where g is an arbitrary substitution of σ that transforms (i, j) to (i_l, j_l). On the strength of the definition of a block, $S_{(i,j)}$ is not a function of the choice of g.

Under certain conditions, the constructed signature generates a desired category, i.e., the equality $\sigma = \sigma_{\varphi\sigma}$ is realized for a class of groups σ.

We shall prove a few auxiliary propositions.

LEMMA 1. Functional signature φ_σ is permissible, satisfies condition (1), and is maximal for σ.

Proof. We shall verify the conditions of signature permissibility (Rudakov's conditions (1)-(5) [3]). The condition $\forall (i, j)\ ((i, j) \in S_{(i, j)})$ is satisfied, since $(i, j) \in w_{(i, j)}$. We shall show that the following condition is satisfied:

$$\forall g \in \sigma \forall (i_1, j_1) \in S \forall k\, (g(s(i_1, j_1, k)) = s(g(i_1, j_1), k)).$$

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Let (i_l, j_l) be the lead pair of the class γ^l to which pairs (i_1, j_1) and $g((i_1, j_1))$ belong. Then if $(i_1, j_1) = g_1((i_l, j_l))$, $g((i_1, j_1)) = g_2((i_l, j_l)) = g(g_1((i_l, j_l)))$, then $s(i_1, j_1, k) = g_1(s(i_l, j_l, k))$, $s(g(i_1, j_1)), k) = g(g_1(s(i_l, j_l, k)))$, i.e., $s(g(i_1, j_1), k) = g(s(i_1, j_1, k))$. The realization of Rudakov's conditions (2), (3), and (5) [3] follows readily from the above. We shall show that condition (2) is satisfied. If $\lambda(i_1, j_1) = \lambda(i_2, j_2)$, there exists a substitution g of σ such that $g(i_1, j_1) = (i_2, j_2)$. Let $(i_1, j_1) = s(i_1, j_1, k)$. Then $s(i_2, j_2, k) = g(s(i_1, j_1, k)) = g(i_1, j_1) = (i_2, j_2)$.

From condition (3) we have

$$(s(i_2, j_2, k) = g(s(i_1, j_1, k)) \Rightarrow (\lambda(s(i_2, j_2, k)) = \lambda(s(i_1, j_1, k))).$$

Analysis for condition (5)

$$(s(s(i_1, j_1, k), r) = s(i_1, j_1, m)) \equiv (g(s(s(i_1, j_1, k), r) = g(s(i_1, j_1, m))) \equiv$$

$$\equiv (s(s(g(i_1, j_1, k), r) = s(g(i_1, j_1), m)) \equiv (s(s(i_1, j_1, k), r) = s(i_2, j_2, m)).$$

The realization of condition (4) is obvious.

The maximality of φ_σ follows directly from the construction.

Since S does not contain functional-similarity classes of cardinality 1 and, by construction, the sets in φ_σ either do not intersect or coincide, condition (1) is satisfied.

The lemma is proved.

We shall show that the following inclusion is valid:

$$\sigma \subset \sigma_\varphi. \tag{2}$$

Let $g \in \sigma$. According to the construction of φ_σ, the condition $\lambda(g(i_1, j_1)) = \lambda(i_2, j_2)$ is satisfied. The realization of the condition $g(s(i_1, j_1, k)) = s(g(i_1, j_1), k)$ was demonstrated in the proof of Lemma 1.

LEMMA 2. Let σ be a subgroup of a symmetric group. If σ_φ coincides with σ for some permissible functional signature φ that satisfies condition (1), signature φ is σ-equivalent to φ_σ.

This lemma allows the examination to be limited to functional signature φ_σ in determining the possibility of coincidence of groups σ_φ and σ.

Proof. We shall consider the set of functional signatures that are σ-equivalent to φ that was described in the theorem [7].

Let $\varphi_{\max}$ be maximal signature for σ that belongs to that set. We shall show that $\varphi_{\max}$ is σ-equivalent to φ_σ. According to the definition of a maximal signature and according to the definition of φ_σ, $S^{\max}_{(i,j)} = w_{(i,j)}$ and $S^\sigma_{(i,j)} = w_{(i,j)}$ for any (i, j). We can convert $S^{\max}_{(i,j)}$ to $S^\sigma_{(i,j)}$ by transformation (1b) described earlier [7]. Since for any (i_1, j_1), $S^{\max}_{(i_1,j_1)} = g(S^{\max}_{(i,j)})$, if $g(i, j) = g(i_1, j_1)$, and $S^\sigma_{(i_1,j_1)} = g(S^\sigma_{(i_1,j_1)})$ by construction, transformation 1 converts all sets that correspond to elements of the given similarity class into one another. That is, $\varphi_{\max}$ can be converted to φ_σ by a superposition of transformations 1, i.e., they are σ-equivalent. Therefore, φ is σ-equivalent to φ_σ.

The lemma is proved.

Thus, $\sigma = \sigma_{\varphi_\sigma}$ if there exists a signature φ such that $\sigma = \sigma_\varphi$ and category Φ is Γ-complete in Σ_φ. A necessary and sufficient coincidence condition is provided by the following lemma.

LEMMA 3. Group σ coincides with group σ_{φ_σ} if and only if the following conditions are satisfied:

$$\forall (i_1, j_1), (i_2, j_2) \in \gamma^m : S_{(i_1, j_1)} = S_{(i_2, j_2)}$$
$$(\exists g \in \sigma : g(S_{(i_1, j_1)}) = S_{(i_2, j_2)};\ \forall (i_3, j_3) \notin S_{(i_1, j_1)}\ g(i_3, j_3) = (i_3, j_3)), \tag{3}$$

$$\forall (i_1, j_1), (i_2, j_2) \in \gamma^m : S_{(i_1, j_1)} \cap S_{(i_2, j_2)} = \varnothing \tag{4}$$
$$(\exists g \in \sigma : g(S_{(i_1, j_1)}) = S_{(i_2, j_2)};\ g(S_{(i_2, j_2)}) = S_{(i_1, j_1)}$$
$$\forall (i_3, j_3) \notin S_{(i_1, j_1)} \cup S_{(i_2, j_2)}\ g(i_3, j_3) = (i_3, j_3)),$$

and the sets are taken over φ_σ.

Proof. If group σ coincides with group σ_{φ_σ}, substitutions are found in group σ that satisfy conditions (3) and (4), since signature φ_σ is permissible and satisfies condition (1), according to earlier Lemma 4 [7]. Necessity is proved.

As has been shown, $\sigma \subseteq \sigma_{\varphi_\sigma}$. Therefore, it is sufficient to show that $\sigma_{\varphi_\sigma} \subseteq \sigma$.

Let us examine an arbitrary substitution g of σ_{φ_σ}. Substitution g transforms any block w into itself or into another block. Let $w^1, \ldots, w^p$ be blocks that g converts into themselves. Then we can represent g as a composition of substitutions $g = h \circ f^1 \circ \ldots \circ f^p$, where f^k is a substitution that transforms block w^k into itself but does not change the other pairs of S and h is a substitution that either leaves all elements of the block unchanged or converts the block to another block. Since g transforms sets into sets, substitutions $f^1, \ldots, f^p$ also transform sets into sets. Therefore, $f^1, \ldots, f^p$ are contained in σ, according to condition (3). We shall examine the effect of substitution h on the set of blocks. Any substitution can be represented as a product of transpositions. Therefore, h can be represented as $h^1 \circ \ldots \circ h^t$, where h^k is a substitution that changes the places of two blocks but leaves in place the pairs that do not belong to those blocks. According to condition (4), $h^1, \ldots, h^t$ also belong to σ. Since $f^1, \ldots, f^p$ and $h^1, \ldots, h^t$ belong to σ, their product g also belongs to σ.

The lemma is proved.

Definition 2. A group σ is a functional group if $\sigma = \sigma_{\varphi_\sigma}$.

Before describing the functional groups, let us introduce a few definitions.

Definition 3. The area of action of a nonunique subgroup g of a symmetric group σ_0 that acts on set S is the set of elements of S that are not stationary with respect to g. For a unique subgroup, we shall assume that any subset of S is an area of action.

Definition 4. Two pairs of S are connected if they belong to the same functional-similarity class or are contained in the same block.

We see that the connectedness relationship is an equivalence relationship and, therefore, specifies the partitioning of S into equivalence classes, which we shall call connectedness domains and denote by the symbol δ with indices.

Definition 5. A cell is the set of elements of a block that are contained in a given functional-similarity class.

Notation: V^{kmn}, where k indicates the connectedness domain, m the block, and n the number of the cell in the block.

We shall formulate a lemma, whose proof will be given below.

LEMMA 4. All blocks that are contained in the same connectedness domain have the same cardinality and consist of the same number of cells of equal cardinality.

Thus, on set S the functional signature and its corresponding functional group specify the following system of subsets: connectedness domains, which are divided into blocks, which consist of cells.

Two more concepts are required to describe the structure of functional groups.

Definition 6. Let $g_1, \ldots, g_n$ be isomorphic subgroups with σ_0 that have nonintersecting areas of action and let f_k be an isomorphism that transforms g_1 into g_k. Then the consistent product of groups $g_1, \ldots, g_n$ is a group consisting of substitutions of the form $a \circ f_1(a) \circ \ldots \circ f_n(a)$, where a runs through group g_1.

Notation: $g_1 \| \ldots \| g_n$. In the figures, a consistent product of groups will be depicted as a rectangle that encloses those groups.

Definition 7. A subgroup σ_0 is a base subgroup if it transforms any element of the area of action into any other and if no substitution of that group has fixed elements in the area of action.

Base groups will be denoted by the character τ with indices. In the figures, base groups will be represented by triangles.

Examples of base groups are cyclic groups that have been generated by a substitution whose order coincides with the cardinality of the area of action and transitive groups without fixed elements.

Now we shall provide an overall survey of the structure of functional group σ.

Functional group σ is direct product of groups $G^1, \ldots, G^p$, whose areas of action are connectedness domains.

Each group G^k is a direct product of groups H^k and Sh^k. Group H^k acts on the set of blocks of a given connectedness domain as a symmetric group. Group Sh^k leaves the blocks in place while changing the places of the pairs within them.

Group Sh^k is a direct product of conjugate in σ_0 subgroups $\sigma^1, \ldots, \sigma^{r_k}$, whose areas of action are blocks (r_k is the number of blocks in connectedness domain δ^k).

Each group σ^m is a consistent product of isomorphic base subgroups whose areas of action are cells of the block on which σ^m acts.

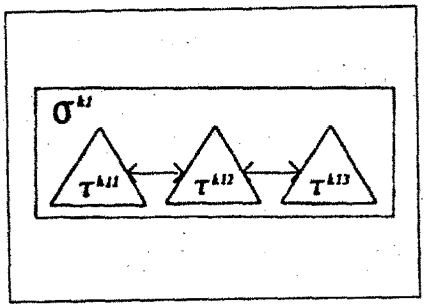

Fig. 1

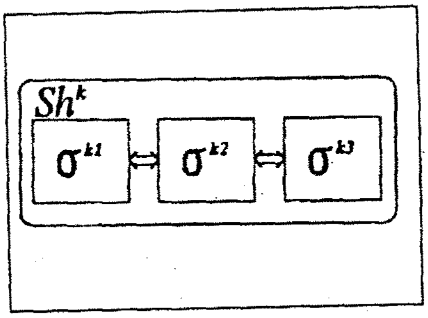

Fig. 2

We now move to a more formal description of the structure of functional groups. We shall first describe a type of substitution group that we shall call an F-group and then show that a group is functional if and only if it is an F-group. Objects that belong to F-groups will be indicated by asterisks.

The F-group to be described is a subgroup of a symmetric group σ_0 that acts on the set of pairs $S = \{(1, 1), \ldots, (q, 1)\}$.

This set is divided into p subsets each of which consists of $r_k t_k z_k$ elements ($k = 1 \ldots p$, r_k, t_k, z_k are arbitrary natural numbers). The subsets will be denoted as δ^k_*. Each of them is divided into r_k subsets $w^{km}{}_*$ of equal cardinality: $|\ w^{km}{}_*\ | = t_k z_k$. Each of subsets $w^{km}{}_*$ is divided into t_k subsets $V^{kmn}{}_*$ of cardinality z_k.

We specify for each k a family of substitutions f^{km}, $m = 2, \ldots, r_k$, that transform $w^{k1}{}_*$ and $w^{km}{}_*$ into one another and leave the remaining elements of S unchanged.

For each $V^{k1n}{}_*$, $k = 1, \ldots, p$, $n = 1, \ldots, t_k$, we specify a base group $\tau^{k1n}{}_*$, where for a fixed k all groups $\tau^{k1n}{}_*$ must be isomorphic.

We define group $\sigma^{k1}{}_*$ as a consistent product $\tau^{k11}_* \parallel \ldots \parallel \tau^{k1t_k}_*$ (Fig. 1). In the figures, single and double lines connect isomorphic and conjugate groups, respectively.

We define groups $\sigma^{k2}_*, \ldots, \sigma^{kr_k}_*$ as follows: $\sigma^{km}_* = f^{km} \sigma^{k1}_* (f^{km})^{-1}$, i.e., as subgroups that are conjugate to $\sigma^{k1}{}_*$ with respect to f^{km}.

Group $Sh^k{}_*$ is defined as a direct product $\prod_{m=1}^{r_k} \sigma^{km}_*$ (Fig. 2). A direct product is represented by an oval in the figures.

Group $H^k{}_*$ is group that has been stretched to the set of substitutions $\{f^{km}\}$. Since f^{km} specifies the transposition of the first and mth blocks, $H^k{}_*$ acts on the set of blocks as a symmetric group. In the figures, $H^k{}_*$ is indicated by a rhombus.

Group $G^k{}_*$ is defined as $Sh^k_* \times H^k_*$ (Fig. 3).

Finally, we define an F-group as a direct product $\prod_{k=1}^{p} G^k_*$ (Fig. 4).

Now we shall show that any functional group is an F-group. First we shall prove Lemma 4.

Proof. Let blocks w^{ki} and w^{kj} be two different blocks from the same connectedness domain. Then the blocks contain elements from the same functional-similarity class. According to Lemma 3, we find that these blocks can be transformed into one another by a substitution of σ. Therefore, all blocks from the same connectedness domain are of equal cardinality.

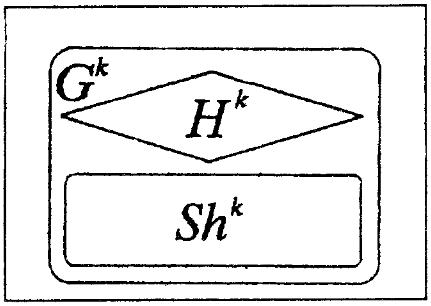

Fig. 3

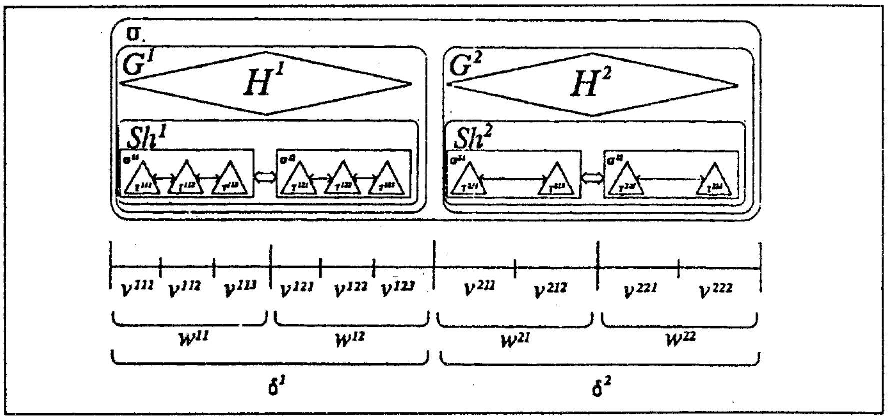

Fig. 4

We shall show that all cells of the same block are of equal cardinality. We shall assume that cell V^{ki1} of block w^{ki} contains t_1 elements and cell V^{ik2} contains t_2 elements, where $t_1 > t_2$. We fix element (i_1, j_1) in V^{ki1}. There exist $t_1 - 1$ substitutions of σ that transform (i_1, j_1) into different elements of V^{ki1}. All of these substitutions must transform block w^{ki} into itself, i.e., transform a fixed element (i_2, j_2) of V^{ki2} into elements of V^{ki2}. Therefore, there exist two substitutions that transform (i_1, j_1) into different elements and (i_2, j_2) into the same elements. But this is inconsistent with the definition of a block. That is, the cardinalities of all blocks are equal. Since any two blocks of the same connectedness domain can be transformed into one another by a substation of σ, the numbers of cells in all blocks of a connectedness domain are the same. The lemma is proved.

LEMMA 5. σ is a direct product of groups whose areas of action are connectedness domains: $\prod_{k=1}^{p} G^k$, $D(G^k) = \delta^k$.

Proof. No substitution of σ takes a pair out of a connectedness domain. As was shown in Lemma 3, any substitution of σ can be represented as $f^1 \circ \ldots \circ f^p \circ h^1 \circ \ldots \circ h^t$, where $h^1, \ldots, h^t$ are transposition of blocks and $f^1, \ldots, f^p$ each transform a corresponding block into itself, leaving the elements of the other blocks unchanged. We select from this product substitutions that refer to each of the connectedness domains. (Each of the substitutions $f^1, \ldots, f^p$ and $h^1, \ldots, h^t$ acts on the elements of only one connectedness domain.) The substitutions whose areas of action do not intersect commute. Thus, group σ is a direct product of groups whose areas of action are contained in different connectedness domains. On the strength of the assumption of the absence of stationary elements, the areas of action of these groups coincide with the connectedness domains. The lemma is proved.

LEMMA 6. 1. A group G^k with area of action δ^k can be represented as a direct product $Sh^k \times H^k$. Group H^k is stretched to the set of transpositions f^{1m}, $m = 2, \ldots, r_k$, which transpose blocks w^{k1} and w^{km} (r_k is the number of blocks in δ_k) and group Sh^k is the group of all substitutions of G^k that leave the blocks unchanged.

2. Group Sh^k can be represented as a direct product $\prod_{m=1}^{r_k} \sigma^{km}$, where σ^{km} is a group that transposes the elements of w^{km} without moving the elements of the other blocks, and all groups σ^{km} are conjugate for a given k.

Proof. Any substitution g of G^k transforms sets into sets (sets are considered according to the functional signature φ_σ).

We fix for each block w^{km} a set S^m. Let $g(S^{m_1}) = S'^{m_2}$ (S'^{m_2} is a set corresponding to w^{km} that does not necessarily coincide with S^{m_2}). According to Lemma 3, there exists a substitution d^{m_2} that transforms S'^{m_2} into S^{m_2}. Therefore, g can be represented as $h \circ (d^1 \circ \ldots \circ d^{r_k})^{-1}$, where h is a substitution that transposes sets $S^1, \ldots, S^{r_k}$.

Group H^k, which is made up of all possible substitutions that transpose $S^1, \ldots, S^{r_k}$, is contained in G^k, according to Lemma 3.

Group Sh^k, by definition, contains all substitutions of G^k that leave the blocks unchanged. A substitution g can be represented as a product of elements of Sh^k and H^k, i.e., $G^k = Sh^k \times H^k$.

It is obvious that any substitution of Sh^k can be represented as $d^1 \circ \ldots \circ d^{r_k}$, where $d^1, \ldots, d^{r_k}$ are substitutions with areas of action $w^{k1}, \ldots, w^{kr_k}$. According to the definition of Sh^k, groups $\sigma^{k1}, \ldots, \sigma^{kr_k}$, which consist of all substitutions that transpose the elements of a given block, are contained in Sh^k. Thus, $Sh^k = \prod_{m=1}^{r_k} \sigma^{km}$. Any two groups σ^{km_1} and σ^{km_2} are conjugate with respect to the element of H^k that transposes sets S^{m_1} and S^{m_2}. The lemma is proved.

LEMMA 7. Group σ^{km} is a consistent product of the base groups τ^{kmn} whose areas of action are cells of V^{kmn}, $n = 1, \ldots, t_k$.

Proof. We define group τ^{kmn} as follows: the area of action of the group is the cell V^{kmn} on whose area of action the substitutions of τ^{kmn} coincide with the substitutions of σ^{km}. Let the mapping $f^n: \tau^{kmn} \to \sigma^{km}$ place a substitution of τ^{kmn} in correspondence with the substitution of σ^{km} that generated it. We shall assume that substitutions g_1 and g_2 of σ^{km} generate the same substitution of τ^{kmn_1} and different substitutions of τ^{kmn_2}. Then there exist pairs (i_1, j_1) of V^{kmn_1} and (i_2, j_2) of V^{kmn_2} such that $g_1(i_1, j_1) \neq g_2(i_2, j_2)$, $g_1(i_2, j_2) = g_2(i_2, j_2)$, and this is in conflict with the definition of a block.

Group τ^{kmn} is a base group, since, according to the definition of a block, none of its nonunique substitutions can have fixed elements, and according to the definition of a functional-similarity class, it contains substitutions that transform any given element of V^{kmn} into another.

All groups τ^{kmn} are isomorphic with σ^{km}; isomorphism is established by the mapping of f^n.

According to the definition of τ^{kmn}, any substitution g of σ^{km} can be represented as $g = (f^1)^{-1}(g) \circ \ldots \circ (f^{t_k})^{-1}(g)$, from which it follows that σ^{km} is a consistent product of groups τ^{kmn}, $n = 1, \ldots, t_k$. The lemma is proved.

It can be concluded on the basis of Lemmas 4-7 that each object of the definition of an F-group corresponds to an identical object that is associated with a fixed group: $\delta^k{}_*$ corresponds to δ^k, $G^k{}_*$ to G^k, etc.

We shall now show that any F-group is a functional group.

LEMMA 8. If σ is an F-group, $\sigma = \sigma_{\varphi_\sigma}$.

Proof. It follows from the definition of an F-group that sets $w^{km}{}_*$ are blocks with respect to σ and $\bigcup_{m=1}^{r_k} V_*^{kmn}$ are functional-similarity classes.

If (i_1, j_1) and (i_2, j_2) belong to the same functional-similarity class (first alternative of Lemma 3), (i_1, j_1) and (i_2, j_2) belong to the same cell $V^{kmn}{}_*$. Then their corresponding sets $S_{(i_1, j_1)}$ and $S_{(i_2, j_2)}$ of functional signature φ_σ coincide in element composition with block $w^{km}{}_*$. On the strength of the definition of $\tau^{kmn}{}_*$, a substitution g is found in $\sigma^{km}{}_*$ that transforms (i_1, j_1) into (i_2, j_2), according to the construction of φ_σ, and $S_{(i_1, j_1)}$ is transformed into $S_{(i_2, j_2)}$. Since $D(\sigma_*^{km}) = w_*^{km}$, the remaining elements of S are unchanged. Thus, σ ($\sigma^{km}{}_*$ is a subgroup of σ) contains a substitution required to satisfy the condition of Lemma 3.

Now let (i_1, j_1) and (i_2, j_2) belong to different blocks. In this case, (i_1, j_1) belongs to cell $V_*^{km_1n}$ and (i_2, j_2) belongs to cell, where $m_1 \neq m_2$. Then their corresponding sets coincide with blocks $w_*^{km_1}$ and $w_*^{km_2}$. A substitution h of $H^k{}_*$ is found that transforms these blocks into one another. With the aid of substitutions s_1 and s_2 of $\sigma_*^{km_1}$ and $\sigma_*^{km_2}$, we obtain a substitution $s_1 \circ h \circ s_2$ that transforms $S_{(i_1, j_1)}$ into $S_{(i_2, j_2)}$ and vice versa. The condition of Lemma 3 is satisfied.

We shall formulate the result obtained in Lemmas 4-8 as a theorem.

THEOREM. In order that for a group σ that does not contain stationary elements there exists a signature φ such that $\sigma = \sigma_\varphi$ and such that category Φ is complete in category Σ_φ, it is necessary and sufficient that σ be an F-group.

This theorem answers the question raised at the beginning of the article.

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