Computing and Visualizing a Graph-Based Decomposition for Non-Manifold Shapes

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Abstract. Modeling and understanding complex non-manifold shapes is a key issue in shape analysis and retrieval. The topological structure of a non-manifold shape can be analyzed through its decomposition into a collection of components with a simpler topology. Here, we consider a decomposition of a non-manifold shape into components which are almost manifolds, and we present a novel graph representation which highlights the non-manifold singularities shared by the components as well as their connectivity relations. We describe an algorithm for computing the decomposition and its associated graph representation. We present a new tool for visualizing the shape decomposition and its graph as an effective support to modeling, analyzing and understanding non-manifold shapes.

1 Introduction

Non-manifold models have been introduced in geometric modeling long time ago. They are relevant in describing the shape of mechanical models, which are usually represented as volumes, surfaces and lines connected together. Informally, a *manifold* (with boundary) M is a compact and connected subset of the Euclidean space for which the neighborhood of each points of M is homeomorphic to an open ball (or to an open half-ball). Shapes, that do not fulfill this property at one or more points, are called *non-manifold*.

Non-manifold shapes are usually discretized as cell or simplicial complexes and arise in several applications, including finite element analysis, computer aided manufacturing, rapid prototyping, reverse engineering, animation. In Computer Aided Design (CAD), non-manifold shapes are usually obtained through an idealization process which consists of operations, such as removal of details, hole removal, or reduction in the dimensionality of some parts. For instance, parts presenting a beam behavior in an object can be replaced with one-dimensional entities, and parts presenting a plate behavior can be replaced by two-dimensional surfaces. This process reduces the complexity of the object, thus resulting in a representation which captures only its essential features.

A natural way to deal with the intrinsic complexity of modeling non-manifold shapes consists of considering a topological decomposition of the shape into manifold or "almost" manifold parts. We consider here a decomposition of a non-manifold shape into what we call *manifold-connected components*, which form a topological super-class of pseudo-manifolds [1]. Further investigation on the properties of such decomposition, that we call an *MC-decomposition*, showed that it is unique and is the discrete counterpart of Whitney stratification used in the differentiable case.

We represent the structure of a non-manifold shape as a hypergraph, that we call the *MC-decomposition graph*, in which the nodes correspond to the MC-components and the arcs describe the connectivity among the components defined by the non-manifold singularities. We have developed an algorithm for computing the MC-decomposition, and its associated graph, based on a new data structure for encoding the discretized input shape, that we have implemented in a library, the *IS library*, for encoding and manipulating simplicial complexes [2].

In our work, we have designed and developed a visualization tool for rendering a segmentation of a shape into parts and its associated graph representation. The tool is completely general and is not tailored to non-manifold shapes, or to the specific MC-decomposition. A beta version of the decomposition software and of the visualization tool can be downloaded from `http://www.disi.unige.it/person/PanozzoD/mc/`.

The MC-decomposition and its associated graph is a very flexible tool for shape analysis, shape matching and retrieval and shape understanding and annotation. We have applied such representation for the computation of topological invariants of a shape, such as the Betti numbers, and for developing a taxonomy for non-manifold shapes [3]. The basis for shape understanding and semantic annotation is extracting and recognizing the so-called *form features* of a shape, such as protrusions or depressions, through-holes or handles. Since form features have been classified in the literature only for manifold shapes, in our previous work, we have extended such classification to non-manifold shapes [4]. The combinatorial structure of the MC-decomposition graph and the topological structure of the MC-components themselves are related to the topology of the original non-manifold shape. Thus, its form features can be extracted through graph-theoretic algorithms applied to the MC-decomposition graph.

The remainder of this paper is organized as follows. In Section 2, we review some related work. In Section 3, we briefly discuss background notions on simplicial complexes. In Section 4, we present the decomposition for non-manifold shapes discretized through simplicial 3-complexes, i.e., the *MC-decomposition*, and a graph-based representation for the MC-decomposition. In Section 5 we describe an algorithm for computing the MC-decomposition and its associated graph. In Section 6, we present the tool we have developed to view the MC-decomposition and its decomposition graph, and we show some results. Finally, in Section 7, we draw some concluding remarks and discuss current and future development of this work.

2 Related Work

Shape analysis is an active research area in geometric and solid modeling, computer vision, and computer graphics. The major approaches to shape analysis are based on computing the decomposition of a shape into simpler parts. Such approaches are either *interior-based*, or *boundary-based* [5]. Interior-based approaches implicitly partition the volume of a shape by describing it as a geometric, or a topological skeleton [6]. Boundary-based methods provide a decomposition of the boundary of an object into parts, by considering local properties of the boundary of the shape, such as critical features or curvature. These latter methods aim at decomposing an object into *meaningful components*, i.e., components which can be perceptually distinguished from the remaining part of the object. Boundary-based methods have been developed in CAD/CAM for extracting form features and produce a boundary-based decomposition of a 3D object guided by geometric, topological and semantic criteria [7].

All shape segmentation and feature extraction algorithms, however, work on manifold shapes. Only few techniques have been proposed in the literature for decomposing the boundary of regular non-manifold 3D shapes [8, 9].

The partition of an analytic variety into analytic manifolds, called a *stratification*, has been studied in mathematics to investigate the properties of such varieties [10]. A stratification expresses the variety as the disjoint union of a locally finite set of analytic manifolds, called *strata*. Pesco et al. [11] introduced the concept of combinatorial stratification as the basis for a data structure for representing non-manifold 3D shapes described by their boundary. The combinatorial stratification for a cell complex is a collection of manifold sub-complexes of different dimensions, the union of which forms the original complex. A combinatorial stratification as discussed in [11], however, is not unique.

3 Background Notions

In this Section, we introduce some background notions on simplicial complexes, which will be used throughout the paper (see [12] for more details).

A Euclidean *simplex* σ of dimension k is the convex hull of $k+1$ linearly independent points in the n-dimensional Euclidean space E^n, $0 \leq k \leq n$. V_σ is the set formed by such points. We simply call a *Euclidean simplex* of dimension k a *k-simplex*. k is called the *dimension* of σ. Any Euclidean p-simplex σ', with $0 \leq p < k$, generated by a set $V_{\sigma'} \subseteq V_\sigma$ of cardinality $p+1 \leq d$, is called a *p-face* of σ. Whenever no ambiguity arises, the dimensionality of σ' can be omitted, and σ' is simply called a *face* of σ. Any face σ' of σ such that $\sigma' \neq \sigma$ is called a *proper face* of σ. A finite collection Σ of Euclidean simplexes forms a *Euclidean simplicial complex* if and only if (i), for each simplex $\sigma \in \Sigma$, all faces of σ belong to Σ, and (ii), for each pair of simplexes σ and σ', either $\sigma \cap \sigma' = \emptyset$ or $\sigma \cap \sigma'$ is a face of both σ and σ'. If d is the maximum of the dimensions of the simplexes in Σ, we call Σ a *d-dimensional simplicial complex*, or a *simplicial d-complex*. In the following, we will restrict our consideration to simplicial 1-, 2- and 3-complexes in the three-dimensional Euclidean space E^3.

The *boundary* of a simplex σ is the set of all proper faces of σ in Σ, while the *star* of σ is the set of simplexes in Σ that have σ as a face. The *link* of σ is the set of all the faces of the simplexes in the star of σ which are not incident into σ. Any simplex σ such that $star(\sigma)$ contains only σ is called a *top* simplex. A simplicial d-complex in which all top simplexes are of dimension d is called *regular*, or *of uniform dimension*. An *h-path* in a simplicial d-complex Σ joining two $(h+1)$-simplexes in Σ, where $h = 0, 1, ..., d-1$, is a path formed by an alternating sequence of h-simplexes and $(h+1)$-simplexes. A complex Σ is said to be *h-connected* if and only if there exists an h-path joining every pair of $(h+1)$-simplexes in Σ. A subset Σ' of Σ is a *sub-complex* if Σ' is a simplicial complex. Any maximal h-connected sub-complex of a d-complex Σ is called an *h-connected* component of Σ.

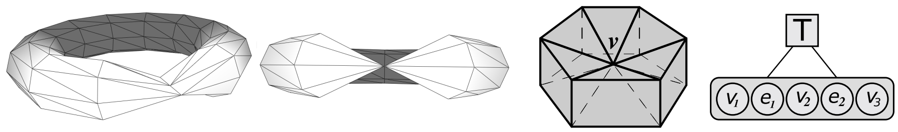
4 The MC-Decomposition into Manifold-Connected Components

In this Section, we describe a decomposition for non-manifold shapes discretized through simplicial 2- and 3-complexes, first introduced in [1], called the *MC-decomposition* and a graph representation for such decomposition.

The non-manifold singularities in the combinatorial representation of a non-manifold shape are characterized by defining non-manifold vertices and edges. A vertex (0-simplex) v in a d-dimensional regular complex Σ, with $d \geq 1$, is a *manifold* vertex if and only if the link of v in Σ is a triangulation of the $(d-1)$-sphere S^{d-1}, or of the (d–1)-disk B^{d-1}. A vertex (0-simplex) v in a 1-dimensional regular complex Σ is a manifold vertex if and only if the link of v consists of one or two vertices. A vertex is called *non-manifold* otherwise.

An edge (1-simplex) e in a regular 3-complex Σ is a *manifold* edge if and only if the link of e in Σ is a triangulation of the 1-sphere S^1, or of the 1-disk B^1. An edge (1-simplex) e in a regular 2-complex Σ is a *manifold* edge if and only if the link of e in Σ consists of one or two vertices. An edge is called *non-manifold* otherwise.

The building blocks of the decomposition are *manifold-connected (MC)* complexes. We consider a regular simplicial d-complex Σ embedded in the three-dimensional Euclidean space, where $d = 1, 2, 3$. In such a complex, we say that a $(d-1)$-simplex σ is a *manifold* simplex if and only if there exist at most two d-simplexes in Σ incident in σ. A $(d-1)$-path such that every $(d-1)$-simplex in the path is a manifold simplex is called a *manifold (d-1)-path*. Thus, we say that two d-simplexes in Σ are *manifold-connected* if and only if there exists a manifold $(d-1)$-path connecting them. Then, we call a regular simplicial d-complex Σ a *manifold-connected* complex if and only if any pair of d-simplexes in Σ are manifold-connected. Figures 1(a) and 1(b) show examples of manifold-connected 2- and 3-complexes, respectively. Note that manifold-connected 2- and 3-complexes may contain both non-manifold vertices and edges. It can be easily seen that a 1-dimensional manifold-connected complex cannot contain either non-manifold vertices or edges.



(a) (b) (c)

Fig. 1. (a) Example of a manifold-connected 2-complex; (b) example of a manifold-connected 3-complex; (c) MC-decomposition graph for the complex in Figure 1(a): non-manifold edges e_1, e_2 and non-manifold vertices v_1, v_2, v_3 define the non-manifold singularity in the pinched torus of Figure 1(a).

A simplicial 3-complex Σ embedded in the three-dimensional Euclidean space can be decomposed into manifold-connected one-, two- and three-dimensional complexes, called *Manifold-Connected (MC) components*. Recall that a subset Σ' of a complex Σ is a *sub-complex* if Σ' is a simplicial complex. Intuitively, a decomposition Δ of Σ is a collection of sub-complexes of Σ, such that the union of the components in Δ is Σ, and any two components Σ_1 and Σ_2 in Δ, if they intersect, intersect at a collection of non-manifold vertices and edges. An *MC-decomposition* is constructively defined by applying the following property: two k-dimensional top simplexes σ_1 and σ_2 belong to the same MC-component if and only if there exists a manifold $(k-1)$-path that connects σ_1 and σ_2 in Σ. It can be proved that the MC-decomposition is unique and that the MC-decomposition is the closest combinatorial counterpart of a Whitney stratification.

The MC-decomposition Δ can be described as a hypergraph $H =< N, A >$, called the *MC-decomposition graph*, in which the nodes correspond to the MC-components in Δ, while the hyperarcs correspond to the non-manifold singularities common to two or more components, or within a single component. The hyperarcs that connect distinct components are defined as follows: any k components $C_1, C_2, \cdots, C_k$ in the MC-decomposition, with $k > 1$, such that the intersection J of all such components is not empty, and J is common only to the k components, defines one or more hyperarcs with extreme nodes in $C_1, C_2, \cdots, C_k$. The intersection of components $C_1, C_2, \cdots, C_k$ consists of isolated non-manifold vertices, or maximal connected 1-complexes formed by non-manifold edges. A hyperarc is a connected component of such intersection. Thus, we classify hyperarcs as *0-hyperarcs*, which consist only of one non-manifold vertex and as *1-hyperarcs*, which are maximal 0-connected 1-complexes formed by non-manifold edges. Figure 2(b) shows the MC-decomposition graph of the simplicial 2-complex depicted in Figure 2(a). The complex is formed by three triangles incident at a common edge e_1 and by a dangling edge C_4 incident at one extreme of e_1. The MC-decomposition graph consists of four nodes that represent the four components, each of which is made of a single top cell, and of two hyperarcs. A 1-hyperarc is associated with vertex v_1 and edge e_1, and a 0-hyperarc is associated with vertex v_2. Since a component C may contain non-manifold singularities, we represent

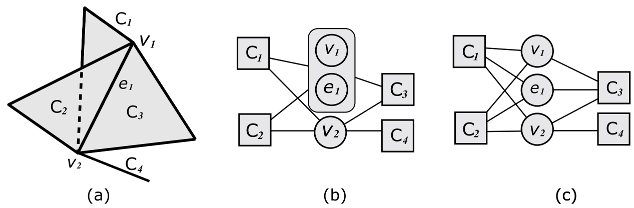


Fig. 2. A simplicial 2-complex (a), its corresponding MC-decomposition graph (b) and the exploded version of the MC-decomposition graph (c).

C in the decomposition graph with a node and with self-loops corresponding to the non-manifold vertices and non-manifold edges. A 0-hyperarc corresponds to a non-manifold vertex belonging to C, while a 1-hyperarc corresponds to a maximal connected 1-complex formed by non-manifold edges and vertices all belonging to C. Figure 1(c) shows the MC-decomposition graph for the pinched torus depicted in Figure 1(a): the graph contains one self-loop corresponding to the non-manifold edges and vertices forming the non-manifold singularity in the shape.

5 Computing the MC-decomposition Graph

Our algorithm for computing the MC-decomposition of a simplicial 3-complex Σ extracts first the maximal connected k-dimensional regular sub-complexes of Σ of dimensions 0, 1 and 2, and then computes the MC-decomposition of each k-dimensional regular sub-complex. To compute the MC-decomposition of a k-dimensional regular complex, we use the property stated above that any pair of manifold simplexes belonging to the same k-dimensional manifold-connected component (for $k=1,2,3$) must be connected through a manifold $(k-1)$-path. This means that every MC-component C can be traversed by following the manifold $(k-1)$-paths connecting the k-simplexes in C. We consider then a graph G in which the nodes are the top k-simplexes, $k = 1,2,3$, and the arcs connect any pair of top k-simplexes which share a manifold $(k-1)$-simplex. The connected components of such a graph are the manifold-connected components in the MC-decomposition.

We compute first an exploded version of the MC-decomposition graph, that we call the *expanded MC-decomposition graph*. In the expanded MC-decomposition graph, that we denote as $H_E = (N_E, A_E)$, the nodes are in one-to-one correspondence with the MC-components, while the hyperarcs are in one-to-one correspondence with the non-manifold vertices and edges. A hyperarc corresponding to a non-manifold vertex v (or to a non-manifold edge e) connects all the MC-components that contain vertex v (or edge e). Figure 2(c) shows the expanded MC-decomposition graph of the simplicial 2-complex depicted in Figure 2(a). A hyperarc is associated with each non-manifold singularity of the complex.

The MC-decomposition graph H is then computed from its expanded version H_E by merging in a single hyperarc connecting components $C_1, C_2, ..., C_q$ all the hyperarcs of G which connect all such components and correspond to non-manifold vertices and edges which form a connected 1-complex. In other words, if we consider the connected components of the 1-complex formed by the non-manifold vertices and edges shared by $C_1, C_2, ..., C_q$, then the hyperarcs in H joining $C_1, C_2, ..., C_q$ are in one-to-one correspondence with such connected components.

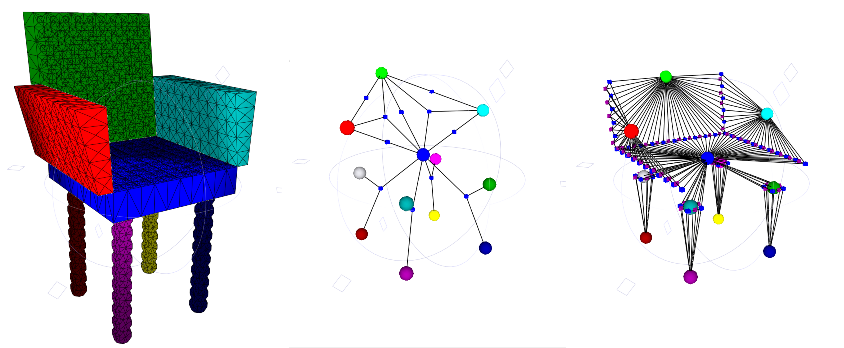
Our implementation of the MC-decomposition algorithm is based on the *IS library*, which implements the *Incidence Simplicial (IS)* data structure together with traversal and update operators. The IS data structure is a new dimension-independent data structure specific for d-dimensional simplicial complexes, that encodes all simplexes in the complex explicitly and uniquely, and some topological relations among such simplexes [2]. We use such information to detect non-manifold singularities in the input complex and to perform an efficient traversal of the complex. By using the IS data structure, the computation of the MC-decomposition graph has a time complexity linear in terms of the number of simplexes in Σ.

6 Visualizing the MC-decomposition and the MC-decomposition Graph

We have developed a tool for visualizing a decomposition of a simplicial complex and its decomposition graph. This tool is called *Graph-Complex Viewer (GCViewer)* and can visualize d-dimensional simplicial complexes, with $d = 1, 2, 3$, embedded in E^3. *GCViewer* can be used as a stand-alone viewer for a simplicial complex, or as a C++ library. *GCViewer* is general and is not tailored to a specific decomposition. Thus, it is intended as a support to the development and analysis of any graph-based representation for discretized shapes. Right now, it is restricted to 3D shapes discretized as simplicial complexes, but it can be easily extended to deal with cellular shape decompositions. The MC-decomposition algorithm, described before, has been developed as a plug-in for *GCViewer*.

GCViewer allows the user to specify a set of graphs, embedded in E^3, and provides a rich set of visualization capabilities to personalize the rendering of both the complex and the graph. The user interface of *GCViewer* allows generating one or more views of the complex. For each view, it is possible to show, hide, or personalize the rendering options of each component and graph that has been defined. In *GCViewer*, we have developed a new technique for an effective visualization of the graph representing the decomposition of a shape, that we have applied in rendering both the MC-decomposition graph and its expanded version. The issue here is that the graphs are not planar. Since the tool should be a support for an effective shape analysis and semantic annotation, the layout of the graph nodes should visually reflect the position of the components in the shape decomposition (in our case, in the MC-decomposition). We have used the Cartesian coordinates of the vertices in each MC-component of the original complex to compute an embedding of the nodes of graph in 3D space. We place each node at the barycenter of its associated component. This greatly improves the readability of both the MC-decomposition graph and of its exploded version by also showing visually the correspondence with the shape decomposition.

Figure 3 depicts a screenshot from *GCViewer* showing the original shape, its MC-decomposition (into twelve MC-components), the MC-decomposition graph and its exploded version. The MC-decomposition is shown in the original shape by assigning different colors to the components. Note that the MC-components are the back, the seat, the two armrests, the four legs and four pieces which connect the legs to the seat.

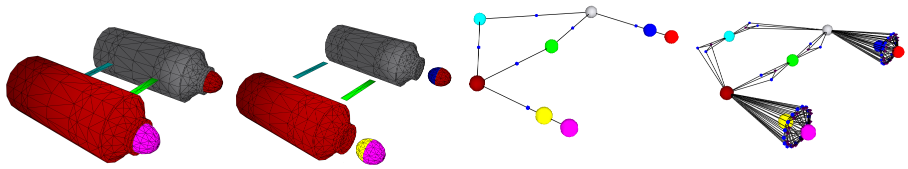


(a) (b) (c)

Fig. 3. A screenshot from *GCViewer*, that shows a complex representing an armchair, highlighting its twelve MC-components (a), its MC-decomposition graph (b) and the exploded version (c).

Figure 4(a) shows a shape formed by two bottles connected by two laminas (2-dimensional MC-components), plus the caps, each of which consists of two MC-components. The two bottles with the two laminas form a 1-cycle in the shape. This is reflected in the cycle in the MC-decomposition graph, shown in Figure 4(c). As shown by this example, there is a relation between the cycles in the graph and the 1-cycles in the original shape which is not, however, a one-to-one correspondence. Not all the cycles in the graph correspond to 1-cycles in the shape, as shown in the example of Figure 3. 1-cycles in the shape that appear as cycles in the MC-decomposition graph are those containing non-manifold singularities. We are currently investigating the relation of the 1-cycles in the shape with the properties of the MC-decomposition graph.

Beta binary versions of the visualization tool and of the MC-decomposition algorithm are available at `http://www.disi.unige.it/person/PanozzoD/mc/`.



(a) (b) (c) (d)

Fig. 4. A complex representing a pair of bottles connected by two laminas (a); an expanded version of the complex that shows its internal structure (b); the corresponding MC-decomposition graph (c) and the corresponding exploded graph (d).

7 Concluding Remarks

We have presented a decomposition for non-manifold shapes into manifold-connected components. We have discussed the MC-decomposition graph as a description of the connectivity structure of the decomposition and we have shown through examples how the combinatorial properties of the MC-decomposition graph are related to the topology of the decomposed shape. We have also described an innovative tool for visualizing the decomposition and its associated graph.

The MC-decomposition and its graph representation are the basis for applications to the analysis, understanding, retrieval and semantic annotation of non-manifold shapes. In our current work, we are using the MC-decomposition as the basis for computing topological invariants of a non-manifold shape, like the Betti numbers. These latter are computed by reconstructing from the MC-decomposition what we call a shell-based decomposition. The shell-based decomposition is obtained by combining together into closed components the MC-components that form a 2-cycle in the shape. Betti numbers are an important topological shape signature to be used for shape classification and retrieval.

Another important application is detecting form features in a non-manifold shape, based on the structure of the single components and on the combinatorial structure of the decomposition. This is a very relevant issue in CAD, where non-manifold shapes are helpful in describing mechanical models, often obtained as the idealization of manifold ones.

In our future work, we plan to use the MC-decomposition as the basis for shape matching and retrieval. This unique topological decomposition can be combined with unique descriptions of the manifold parts, like the Reeb graph, thus forming the basis for a two-level shape recognition process. Moreover, an important issue is to study how the MC-decomposition is affected by updating the underlying shape and its simplicial discretization. In this context, we plan to analyze and classify operators for modifying a non-manifold shape and to develop algorithms for efficiently updating the decomposition based on such operators.

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