Generalized Motorcycle Graphs for Imperfect Quad-Dominant Meshes

Supplemental Material

1 OPTIMIZATION OF MOTORCYCLE SPAWNING

In this section, we explain how to optimize for the starting positions and exit paths for spawning motorcycles from irregular fenced regions.

At this point, potential paths have been traced that connect possible exit points with the interior vertices of the fenced region. The goal is to select a subset of v paths (v being the fenced region's valence) from these candidates that represent the final exit paths. For this, we prefer paths that are straight and where the starting point is centered inside the fenced region.

We evaluate the quality of a starting point q and the final exit paths $P = (p_1, p_2, ...p_v)$, where the p_i are sequences of edges that lead from the starting point to the fenced region's border, with the following energy:

$$E(q, P) = \omega_s E_s(P) + \omega_d E_d(P) + \omega_c E_c(r), \tag{1}$$

where $E_s(P)$ measures the total straightness of all paths, $E_d(P)$ measures how uniformly the paths are distributed around the starting point, and $E_c(q)$ measures how centered the representative is in the fenced region. The weights ω_s, ω_d, and ω_c balance between the energies. In our experiments, we used $\omega_s = 1$, $\omega_d = 1$, $\omega_c = 0.2v$.

The straightness energy is the summed straightness deviation (measured as an angle in the tangent plane at every vertex) of all paths:

$$E_s(P) = \sum_{p \in P} \sum_{i=1}^{|p|} \sphericalangle_{n_{\text{vert}(p,\, i+1)}}(p[i], p[i+1]), \tag{2}$$

where $\text{vert}(i, p)$ is the i-th vertex of path p, n_v is the normal corresponding to vertex v, and $p[i]$ is the i-th edge of path p.

The distribution energy measures the deviation from the angle between two successive paths at the starting point from 90° (measured in 3D space) because the respective corner will be mapped to a right angle in parameter space:

$$E_d(P) = \sum_{i=1}^{v} \left(\sphericalangle(p_i, p_{i+1}) - \frac{\pi}{2} \right)^2 \tag{3}$$

Finally, the centeredness energy measures the distance of the starting point from the fence, which we calculate with a BFS:

$$E_c(q) = -\text{distanceFromFence}(q) \tag{4}$$

We evaluate the best exit paths for every vertex in the fenced region separately and choose the best vertex as the motorcycle starting point. Then, the task is to find the best subset from a set of path candidates pc_i (the initially traced paths). Calculating the best subset for a single vertex can be done efficiently with a Dynamic Program, observing that E_c is constant, the inner terms of E_s are unary energy terms, and the inner terms of E_d are pair-wise energy terms. We will abbreviate:

$$E_{s_i} := \sum_{j=1}^{|pc_i|} \sphericalangle_{n_{\text{vert}(pc_i,\, j+1)}}(pc_i[j], pc_i[j+1]) \tag{5}$$

$$E_{d_{(i,j)}} := \left(\sphericalangle(pc_i, pc_j) - \frac{\pi}{2} \right)^2 \tag{6}$$

Then, we define the partial cost function $C(o, l, f)$, which returns the best possible energy for exit paths for orientations 0 through o having first path pc_f and last path pc_l. The cost of the best exit paths for the examined vertex is then $\arg\min_{l,f} C(v-1, l, f)$. The partial cost function C can be initialized from all path candidates corresponding to edges of orientation zero with:

$$C(0, i, i) := E_{s_i} \quad \forall\, i : \text{orientation}(pc_i) = 0 \tag{7}$$

The cost function can then be propagated in a DP-way with the following update:

$$C(o, i, f) = \infty \quad \forall\, i : \text{orientation}(pc_i) \neq o \tag{8}$$

$$C(o, i, f) = E_{s_i} + \min_{j \in V} C(o-1, j, f) + E_{d_{(j,i)}} + \begin{cases} E_{d_{(i,f)}} & o = v-1 \\ 0 & \text{else} \end{cases} \tag{9}$$

The set of valid predecessors V to check are those whose entry points lie between those of the first and the i-th path candidate in counter-clockwise direction and that do not intersect with the i-th path candidate. This propagation can be done efficiently by sorting the path candidates by their entry points and grouping them by the orientation of the respective edge.

After the entire function has been calculated, the best paths can be derived by back-propagation.

2 PATCH SPLITTING

In this section, we present how we optimize for distortion-minimizing cuts to split non-rectangular patches like the one shown in Figure 1, allowing an integer scale multiplier at the resulting cut. Our splitting procedure can be applied to each dimension independently. In the following, we will only consider a single dimension (the left-right direction with top-down cuts). At this point, parallel motorcycles that have been used to determine target parametric arc lengths are available. We use them both for determining the thickness of a patch as a discrete sampling (reflected by the length of a motorcycle) and as possible cut candidates.

2.1 Problem Formulation

In order to evaluate isometric distortion, we introduce a variable height h_i for every segment of the patch (between two successive cut candidates, see Figure 1). At the same time, we can measure the lengths l_i of all motorcycles, either in a topological or a geometric sense. The isometric distortion is then the deviation of the segment height with respect to both incident motorcycle lengths:

$$D(h) = \sum_{i=1}^{n} (h_i - l_i)^2 + (h_i - l_{i+1})^2, \tag{10}$$

where n is the number of segments.

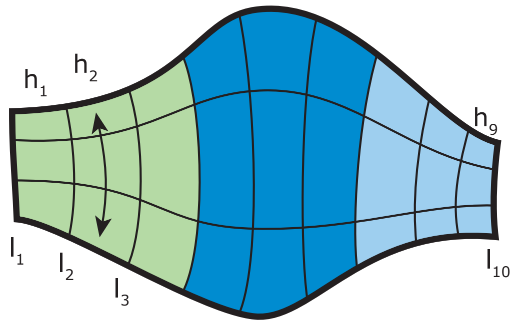

Fig. 1. We split strongly non-rectangular patches like this example patch by considering straight motorcycles between two opposite sides of the patch having lengths l_i as cut candidates. We minimize isometric distortion by allowing power-of-2 multipliers between the segment heights h_i. A possible split is shown as colored segments.

To incorporate the integer constraint for every cut multiplier, we express the heights of all segments as a multiple of the first segment's height: $h_i = m_i \cdot h_1$, where the $m_i \in \left\{\frac{1}{2}, 1, 2\right\}$ are the respective multipliers.

$$D(h_1, m) = (h_1 - l_1)^2 + (h_1 - l_2)^2 + \sum_{i=2}^{n} (m_i \cdot h_1 - l_i)^2 + (m_i \cdot h_1 - l_{i+1})^2 \tag{11}$$

We optimize the multipliers to minimize the above distortion measure and extract connected components of equal multipliers. Finally, we realize the cuts represented by motorcycles between these connected components. If no cuts are necessary, all resulting multipliers will be 1.

To improve robustness of this approach, we perform Laplacian smoothing on the motorcycle length l_i and filter out small connected components with fewer than four segments.

2.2 Solution

Deriving D with respect to one of the multipliers gives the following partial derivative:

$$\frac{\partial D(h_1, m)}{\partial m_i} = 2h_1 \left(2m_i h_1 - l_i - l_{i+1}\right), \tag{12}$$

yielding the following stationary point:

$$m_i = \frac{l_i + l_{i+1}}{2h_1} \tag{13}$$

Intuitively, this means that the multipliers give enough freedom to the objective function such that they could be optimized independently in a continuous setting.

To incorporate the integer constraints, we use an iterative rounding strategy. That means that we solve the optimization problem in a continuous setting and iteratively round one multiplier to one of the valid multipliers. We choose the multiplier whose rounding results in the least energy increase. To evaluate the energy increase, it is also necessary to calculate the optimal h_1. Given the partitioning of the multipliers in fixed multipliers F (those that have been rounded before and stay fixed) and unfixed U, the partial derivative with respect to the first segment's height is:

$$\frac{\partial D(h_1, m)}{\partial h_1} = 4h_1 \cdot \left(1 + \sum_{i \in F} m_i^2\right) - 2\left(l_1 + l_2 + \sum_{i \in F} m_i \cdot (l_i + l_{i+1})\right), \tag{14}$$

yielding the stationary point

$$h_1 = \frac{l_1 + l_2 + \sum_{i \in F} m_i \cdot (l_i + l_{i+1})}{2h_1 \cdot \left(1 + \sum_{i \in F} m_i^2\right)} \tag{15}$$

This optimal height can be calculated incrementally as more multipliers are fixed. This allows to evaluate the change in energy for every possible rounded value of unfixed multipliers. Following the iterative rounding strategy will therefore efficiently result in a local minimizer of the distortion energy.