Sixteenth Northeast Probability Seminar

Thursday and Friday November 15th and 16th, 2018
This conference is supported by the NSF

1 Thursday, Nov. 15th

1.1 Morning Session

Location: NYU Kimmel Center, Room 912/914

9:00 am – 10:00 am Registration, light refreshments

10:00 am – 11:00 am **Rick Durrett** (Duke)

"The contact process on random graphs and trees"

11:00am – 11:30am Refreshments

11:30 am – 12:30 am **Tuca Auffinger** (Northwestern)

"Energy landscape of mean field spin glasses"

12:30 am – 2:15 pm **Lunch break**

1.2 Afternoon Session

2:15 pm **Moumanti Podder** (University of Washington): *Finiteness of Galton-Watson trees and EMSO logic*
2:30 pm **Fan Yang** (UCLA): *Comparison methods in random matrix theory*
2:45 pm **Liying Li** (NYU) *Thermodynamic Limit for Directed Polymers and Stationary Solutions of the Burgers Equation*
3:00 pm **Erik Bates** (Stanford) *Complete path localization of polymers in Gaussian disorder*

3:15 pm – 3:30 pm **Break**

3:30 pm **Guillaume Dubach** (NYU) *Powers of Ginibre Eigenvalues*
3:45 pm **Andrey Sarantsev** (University of Nevada, Reno) *TBA*
4:00 pm **Hyunchul Park** (SUNY New Paltz) *Small time asymptotics of spectral heat contents for subordinate killed Brownian motions related to isotropic α-stable processes*
4:15 pm **Marcus Michelen** (University of Pennsylvania) *Central Limit Theorems based on the Location of Roots of Probability Generating Functions*

4:45 pm – 5:45 pm **Reception** (Courant Institute, 13th Floor Lounge)

6:00 pm **Conference Dinner**

2 Friday, Nov. 16th

2.1 Morning Session

Location: Warren Weaver Hall/Courant Institute, Room 109

10:00 am – 11:00 am **Nalini Anantharaman** (IRMA Strasbourg)

"Quantum ergodicity on graphs : from spectral to spatial delocalization"

11:00am – 11:30am Refreshments

11:30 am – 12:30 am **Roland Bauerschmidt** (Cambridge)

"Spectral gap critical exponent for Glauber dynamics of hierarchical spin models"

12:30 pm – 2:00 pm **Lunch**

2.2 Afternoon Session

2:00 pm **Lingjiong Zhu** (Florida State University) *Global Convergence of Stochastic Gradient Hamiltonian Monte Carlo for Non-Convex Stochastic Optimization*
2:15 pm **Sanchit Chaturvedi** (Stanford) *A percolation type growth model on the continuum*
2:30 pm **Julia Gaudio** (MIT) *Attracting Random Walks*
2:45 pm **Jeremiah Birrell** (University of Massachusetts Amherst) *Langevin equations in the small-mass limit: Phase space homogenization and higher order approximations*
3:00 pm **Berend Coster** (UConn) *Log-Sobolev inequality for the Wright-Fisher diffusion*

3:15 pm – 3:30 pm **Break**

3:30 pm **Somabha Mukherjee** (University of Pennsylvania) *Limiting Distributions and Large Deviations of Motif Counts in Randomly Colored Graphs*
3:45 pm **Xufan Zhang** (Brown University) *The domino shuffling height process and its hydrodynamic limit*
4:00 pm **Dan Han** (University of North Carolina at Charlotte) *Population Dynamics with Immigration on lattice: steady state and its stability with respect to the perturbations*
4:15 pm **George Kerchev** (Georgia Institute of Technology) *On the rate of convergence for the length of the longest common subsequences in hidden Markov models*
4:30 pm **Wenjian Liu** (CUNY QCC) *Big Data Information Reconstruction on the Infinite Communication Tree Network*

3 Abstracts of the Main Talks

3.1 Rick Durrett, Duke: *The contact process on random graphs and trees*

The key to our investigation is an improved (and in a sense sharp) understanding of the survival time of the contact process on star graphs. Using these results, we show that for the contact process on Galton-Watson trees, when the offspring distribution (i) is subexponential the critical value for local survival $\lambda_2 = 0$ and (ii) when it is geometric(p) we have $\lambda_2 \le C_p$, where the C_p are much smaller than previous estimates. We also study the critical value $\lambda_c(n)$ for "prolonged persistence" on graphs with n vertices generated by the configuration model. In the case of power law and stretched exponential distributions where it is known $\lambda_c(n) \to 0$ we give estimates on the rate of convergence. Physicists tell us that $\lambda_c(n) \sim 1/\Lambda(n)$ where $\Lambda(n)$ is the maximum eigenvalue of the adjacency matrix. Our results show that this is not correct.

3.2 Tuca Auffinger, Northwestern: *Energy landscape of mean field spin glasses*

3.3 Nalini Anantharaman, IRMA Strasbourg: *Quantum ergodicity on graphs : from spectral to spatial delocalization*

We prove a quantum-ergodicity theorem on large graphs, for eigenfunctions of Schr dinger operators in a very general setting. We consider a sequence of finite graphs endowed with discrete Schr odinger operators, assumed to have a local weak limit. We assume that our graphs have few short loops, in other words that the limit model is a random rooted tree endowed with a random discrete Schr odinger operator. We show that absolutely continuous spectrum for the infinite model, reinforced by a good control of the moments of the Green function, imply "quantum ergodicity", a form of spatial delocalization for eigenfunctions of the finite graphs approximating the tree.

3.4 Roland Bauerschmidt, Cambridge: *Spectral gap critical exponent for Glauber dynamics of hierarchical spin models*

We develop a renormalisation group approach to deriving the asymptotics of the spectral gap of the generator of Glauber type dynamics of spin systems at and near a critical point. In our approach, we derive a spectral gap inequality, or more generally a Brascamp–Lieb inequality, for the measure recursively in terms of spectral gap or Brascamp–Lieb inequalities for a sequence of renormalised measures. We apply our method to hierarchical versions of the 4-dimensional n-component $|\varphi|^4$ model at the critical point and its approach from the high temperature side, and the 2-dimensional Sine–Gordon and the Discrete Gaussian models in the rough phase (Kosterlitz–Thouless phase). For these models, we show that the spectral gap decays polynomially like the spectral gap of the dynamics of a free field (with a logarithmic correction for the $|\varphi|^4$ model), the scaling limit of these models in equilibrium. This is joint work with Thierry Bodineau.

4 Abstracts of the Talks of Junior Participants

4.1 Moumanti Podder, University of Washington: *Finiteness of Galton-Watson trees and EMSO logic*

It is straightforward to show that infiniteness of rooted trees is expressible in the existential monadic second order (EMSO) language. We show that finiteness, the negation of infiniteness, is not expressible in EMSO. Moreover, we show that it is not possible to throw away a measure-0 subset of rooted infinite trees such that on the complement, finiteness is expressible as an EMSO sentence. This shows that finiteness is not *almost surely expressible* as an EMSO.

4.2 Fan Yang, UCLA: *Comparison methods in random matrix theory*

Comparison method is a powerful tool in random matrix theory. In this talk, we illustrate two comparison methods, the Lindeberg replacement argument and a continuous self-consistent comparison, by showing the proof for the edge universality of separable covariance matrices of the form $A^{1/2}XBX^*A^{1/2}$. Here X is a random matrix with i.i.d. entries and A, B are non-negative definite symmetric covariance matrices. The comparison methods are mainly used to deal with the difficulties that the X entries have heavy tails and A, B are both non-diagonal which give the correlations of the data.

4.3 Liying Li, NYU: *Thermodynamic Limit for Directed Polymers and Stationary Solutions of the Burgers Equation*

We consider Burgers Equation on the real line with stationary random kick-forcing. We will show the inviscid limit of the viscous global solution is the inviscid global solution. This can also be interpreted as the zero-temperature limit for infinite volume polymer measures.

4.4 Erik Bates, Stanford University: *Complete path localization of polymers in Gaussian disorder*

A polymer measure is a probability on random walk paths, where each path's probability is proportional to the product of weights along its vertices. The weights are i.i.d. in space and time, and (in this talk) log-normals, where the log-variance depends on a temperature parameter. If this variance is large, which corresponds to low temperature, the model exhibits localization. That is, the usual diffusive behavior of random walks is replaced by "tightness" of the polymer measures. Formerly this phenomenon has been best understood in terms of the endpoint of the path. In this talk, a new type of result will be described concerning localization of the full path. (Joint work with Sourav Chatterjee)

4.5 Guillaume Dubach, NYU: *Powers of Ginibre Eigenvalues*

Complex Ginibre matrices are random matrices with i.i.d. complex Gaussian entries. It is well known that their eigenvalues exhibit quadratic repulsion, and that their empirical distribution converges to the circular law (uniform distribution on the unit disk) when properly scaled. Although these eigenvalues are strongly correlated, their images under power maps can be analyzed as the superposition of several independent blocks with explicit joint densities. The proof technique that we will present extends to other distributions, and allows to recover Kostlan's theorem : the radii of complex Ginibre eigenvalues are distributed like independent points.

4.6 Andrey Sarantsev, University of Nevada, Reno: *TBA*

4.7 Hyunchul Park, SUNY New Paltz: *Small time asymptotics of spectral heat contents for subordinate killed Brownian motions related to isotropic α-stable processes*

4.8 Marcus Michelen, The University of Pennsylvania: *Central Limit Theorems based on the Location of Roots of Probability Generating Functions*

4.9 Lingjiong Zhu, Florida State University: *Global Convergence of Stochastic Gradient Hamiltonian Monte Carlo for Non-Convex Stochastic Optimization*

Stochastic gradient Hamiltonian Monte Carlo (SGHMC) is a variant of stochastic gradient with momentum where a controlled and properly scaled Gaussian noise is added to the stochastic gradients to steer the iterates towards a global minimum. Many works reported its empirical success in practice for solving stochastic non-convex optimization problems, in particular it has been observed to outperform overdamped Langevin Monte Carlo-based methods such as stochastic gradient Langevin dynamics (SGLD) in many applications. In this work, we provide finite-time performance bounds for the global convergence of SGHMC for solving stochastic non-convex optimization problems with explicit constants. Our results lead to non-asymptotic guarantees for both population and empirical risk minimization problems. For a fixed target accuracy level, we obtain iteration complexity bounds for SGHMC that can be tighter than those for SGLD up to a square root factor. These results show that acceleration with momentum is possible in the context of non-convex optimization algorithms. This is based on the joint work with Xuefeng Gao and Mert Gurbuzbalaban.

4.10 Sanchit Chaturvedi, Stanford: *A percolation type growth model on the continuum*

4.11 Julia Gaudio, MIT: *Attracting Random Walks*

We introduce the Attracting Random Walks model, which is an attractive interacting particle system. In the model, particles move between adjacent vertices of a graph G, with transition probabilities that depend positively on particle counts at neighboring vertices. From an applied standpoint, the model captures systems in which quantity is attractive.

We mention some possible applied areas, but the focus is on developing the properties of the associated Markov chain. The phase transition of the stationary distribution is of particular interest. When G is the complete graph, the model is a projection of the Potts model, whose phase transition is known. We demonstrate the existence of phase transition for general graphs. (Joint work with David Gamarnik, Reza Gheissari, Patrick Jaillet, Eyal Lubetzky, Yuval Peres, and Yury Polyanskiy)

4.12 Jeremiah Birrell, University of Massachusetts Amherst: *Langevin equations in the small-mass limit: Phase space homogenization and higher order approximations*

I will discuss the dynamics of an inertial particle, coupled to forcing, dissipation, and noise, in the small-mass limit. Specifically, the following aspects will be addressed: Convergence of the joint distribution of position and (scaled) velocity degrees of freedom, including instantaneous pointwise equilibration of the (scaled) velocity; Entropy production and the entropy anomaly; Derivation of a hierarchy of homogenized equations for the position degrees of freedom that provides approximations accurate to any desired order in the mass.

4.13 Berend Coster, UConn: *Log-Sobolev inequality for the Wright-Fisher diffusion*

We prove a Log-Sobolev inequality for the one-dimensional Wright-Fisher diffusion by proving a Γ_2 lower bound for this diffusion. The result is extended to the two-dimensional case.

4.14 Somabha Mukherjee, University of Pennsylvania: *Limiting Distributions and Large Deviations of Motif Counts in Randomly Colored Graphs*

Suppose that each of the n vertices of a graph G_n is colored, independently of the others, using one of c_n many colors, chosen uniformly at random. Let S_n denote the number of monochromatic copies of a fixed connected motif in G_n, and T_n denote the number of copies of a fixed connected motif in G_n with all vertices of a particular color. We show that if $\mathbb{E}(S_n)$ and $\mathrm{Var}(S_n)$ converge to the same positive number x, then S_n has a limiting Poisson(x) distribution. We also derive the limiting distribution of T_n when the base motif is the edge, under certain assumptions on the graph G_n. Additionally, we show that T_n, for arbitrary (but fixed) motifs, satisfies a large deviation principle, and in this aspect, we demonstrate some interesting replica symmetry and replica symmetry breaking phenomena. This talk is a combination of my works with Bhaswar Bhattacharya, Dept. of Statistics, UPenn, and Sumit Mukherjee, Dept. of Statistics, Columbia University.

4.15 Xufan Zhang, Brown University: *The domino shuffling height process and its hydrodynamic limit*

The famous domino shuffling algorithm was invented to generate the domino tilings on the Aztec Diamond. Using the domino height function, we view the domino shuffling procedure as a discrete-time random 2D height process. The hydrodynamic limit from an arbitrary continuous profile is deduced to be the unique viscosity solution of a Hamilton-Jacobi equation $u_t + H(u_x) = 0$, where the determinant of the Hessian of H is negative everywhere. It seems that our result is the first example in $d > 1$ where such a full hydrodynamic limit with a nonconvex Hamiltonian can be obtained for a discrete system. We also define the shuffling height process for more general periodic dimer models, where we expect similar results to hold.

4.16 Dan Han, University of North Carolina at Charlotte: *Population Dynamics with Immigration on lattice: steady state and its stability with respect to the perturbations*

Consider the lattice population model with immigration, i.e. the particle filed $n(t,x)$, which evolution includes the random walk generated by discrete Laplace operator with birth rate β, mortality μ and immigration rate k. If β is less than μ, $k > 0$, then one can prove the existence of the steady state. Our goal is to study the stability of this model with respect to different perturbations of the parameter: μ, β, k, which now can be random functions of site x. In contrast to the similar so called contact model (where $\mu = \beta$, $k = 0$), which has also steady state but it is not Lyapunov stable. Steady state in our model is stable in Lyapunov sense if the perturbations are small enough.

4.17 George Kerchev, Georgia Institute of Technology: *On the rate of convergence for the length of the longest common subsequences in hidden Markov models*

Let $(X,Y) = (X_n,Y_n)_{n\geq 1}$ be the output process generated by a hidden chain $Z = (Z_n)_{n\geq 1}$, where Z is a finite state, aperiodic, time homogeneous, and irreducible Markov chain. Let LC_n be the length of the longest common subsequences of $X_1,\dots,X_n$ and $Y_1,\dots,Y_n$. Under a mixing hypothesis, a rate of convergence result is obtained for $\mathbb{E}[LC_n]/n$.

4.18 Wenjian Liu, City University of New York (QCC): *Big Data Information Reconstruction on the Infinite Communication Tree Network*

The information reconstruction problem on the infinite communication tree network, is to collect and analyze massive data samples at the nth level of the tree to identify whether there is non-vanishing information of the root, as n goes to infinity. Although it has been studied in numerous contexts such as information theory, genetics and statistical physics, the existing literatures with rigorous reconstruction thresholds established are very limited. In this project, inspired by a classical Deoxyribonucleic acid (DNA) evolution model, the F81 model, and also taking into consideration of the Chargaff's second parity rule by allowing the existence of a guanine cytosine content bias, we will study the noise channel in terms of a 4×4 asymmetric probability transition matrix with community effects. The corresponding information reconstruction problem in molecular phylogenetics is explored, by means of refined analyses of moment recursion, in-depth concentration estimates, and thorough investigations on an asymptotic 4-dimensional nonlinear second order dynamical system. We will establish the threshold of the reconstruction under the unequal base frequencies of adenine and thymine.