Asymptotically sharp Korn-type inequalities in the mathematical analysis of buckling of cylindrical shells

Abstract

The buckling stress of a circular cylindrical shell dramatically disagrees with its theoretical value. In mechanics this is explained by the extreme sensitivity of buckling to imperfections of shape and load. A relatively new mathematically rigorous theory of “near-flip buckling” is applied to this problem in an attempt to uncover theoretical mechanisms through which the effect of initial imperfections are amplified. The theory of near-flip buckling gives the expression for the buckling load in terms of the best constants in Korn and Korn-type inequalities. This theory predicts “scaling instability” under small changes in applied load, whereby the shell’s buckling stress changes its scaling exponent as a function of shell’s thickness due to different scaling of constants in different Korn-type inequalities. The infinitesimal changes in shape, by contrast, cause a finite jump in the scaling of the constant in the Korn inequality.