CORRECTION SHEET
AN INTRODUCTION TO RANDOM MATRICES
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This list collects the most current list of corrections for the above book. Items marked by $*$ are important, that is, are more than typos. We thank the following people for their comments: Florent Benaych-Georges, Jun Chen, Amir Dembo, Benjamin Fahs, Svante Janson, Toby Johnson, Achim Klenke, Mylène Maïda, Ron Peled, Edouard Maurel Segala, Alain Rouault.

(1) Page 10, line 2, replace $2z$ in the denominator by 2.
(2) $*$ Page 13, line 23, replace "$\ell(w)-1$ edges" by "at most $\ell(w)-1$ edges".
(3) Page 14, line 14, replace "t-words" by "N-words".
(4) $*$ Page 15, line 22 and 30, replace "the next-to-last letter of w_{i-1}" by "the entry preceding the first occurence of the last letter of w_{i-1}".
(5) Page 18, (2.1.23), replace in exponent "a" by "$a_{\mathbf{i},\mathbf{i}'}$".
(6) Page 21, line -5, replace "permutations" by "permutation matrices".
(7) Page 22, line 6, replace "maximum"' by "maximizer"'.
(8) Page 23, line 1, replace "chose" by "choose". line 3, add a missing $|$ after $f(y)$. Theorem 2.1.22, the condition $r_k \le k^{Ck}$ is needed only for integers $k \ge 2$.
(9) Page 26,line -5, replace "sequences" by "sentences". In caption of figure 2.1.3, replace $[x_1, x_2]$ by $[b, c]$.
(10) page 29, Exercise 2.1.30, in first display replace $\le C$ by $< \infty$. In part (a) add that $\|z\|_2 = 1$. In part (2.1.37), replace in the right hand side z^T by $(z - z_i)^T$.
(11) Page 32, line 4, replace "fail" by "fails".
(12) Page 36, line -3, replace $T_{\mathbf{i}}^N$ by $\bar{T}_{\mathbf{i}}^N$.
(13) Page 37, (2.2.4), replace $g_w(1,1)$ by $g_w(1,1)^{k/2}$.
(14) Page 39, line 3, replace $\mathbb{R}^{N(N+1)}$ by $\mathbb{R}^{N^2}$.
(15) Page 40, line -2, replace $G \wedge (-1/\epsilon) \vee (1/\epsilon)$ by $G \wedge (1/\epsilon) \vee (-1/\epsilon)$.
(16) Page 41, line 3, replace $= M \ldots$ by $\le \sqrt{M} \ldots$. In (2.3.8), replace E in left side by E_P.
(17) Page 49, Equation (2.4.14), replace $S_\mu(\lambda + \epsilon)$ by $S_\mu(\lambda + \mathrm{i}\epsilon)$.
(18) Page 59 line -4, replace $\Delta(x)^{2c}$ by $|\Delta(x)|^{2c}$.
(19) Page 73, in (2.6.10), replace $>$ by $\ge$.
(20) Page 75, lines 17-19: erase the part of the sentence "with the strict inequality ... is nontrivial".
(21) Page 76, line -7, Page 77, lines 6 and 17, replace $Z_N^{\beta,V}$ by $Z_{V,\beta}^N$.
(22) Page 78 line -4, -2, -1, page 79 line 3, replace ν by μ.
(23) Page 79 line 7, replace ϵ by δ.
(24) Page 79, line -6, display, replace "$\lim_{N\to\infty}$" by "$\lim_{\delta\to 0}$".
(25) Page 79 line -4, replace $\lambda_i < \lambda_{i-1}$ by $\lambda_i < \lambda_{i+1}$.
(26) Page 80, lines 14, 17, replace $Z_{\beta,V}^N$ by $Z_{V,\beta}^N$.

(27) Page 81, line 7, replace (2.4.6) by (2.4.7).
(28) $*$ Page 81, Assumption 2.6.5: add the assumption that either $J_\beta^V(\cdot)$ achieves its minimum at x^* only, or that under $P^{N-1}_{NV/(N-1),\beta}$, the top eigenvalue converges in probability to x^*. In the former case, one need to use Jensen's inequality in order to show that $J_V^\beta(\cdot) \geq 0$.
(29) $*$ Page 81, display in Theorem 2.6.6 has a sign error, and should read

$$J_\beta^V(x) = \begin{cases} -\beta \int \log|x-y| \sigma_\beta^V(dy) + V(x) - \alpha_{V,\beta} & \text{if } x \geq x^*\,, \\ \infty & \text{otherwise}\,, \end{cases}$$

(30) Page 87, line 14, Pastur *and* co-workers.
(31) Page 88, line -7. add "entries" before "i.i.d.".
(32) Page 91, line 6, replace $\sqrt{n}$ by $\sqrt{N}$, and in (3.1.2), replace σ by $\sigma(t)$.
(33) Page 93, line -4, replace Corollary 3.1.5 by Theorem 3.1.5.
(34) Page 97, equation (3.2.10) up to page 98, 2 lines above Lemma 3.2.4: replace $\tilde{C}_{p,N}$ by $\hat{C}_{p,N} = (N-p)!\tilde{C}_{p,N}$ throughout.
(35) Page 103, "Proof of Lemma 2.1.7" should be replaced by "Proof of Lemma 2.1.6".
(36) Page 105, Exercise 3.3.4, line -2: replace x^{2k} by $(x^{2k} - x^k y^k)$.
(37) Page 106, line 2, replace $K(x,y)$ by $K^{(N)}(x,y)$. Line 6, replace Section 3.2.1 by Section 3.2.2.
(38) Page 110, line -3, replace X^{2p} by $X \times X$.
(39) Page 113, (3.4.25), $+\mathrm{tr}$ in the first line of the display should be $-\mathrm{tr}$.
(40) Page 116, line 2 of proof of Theorem 3.5.3, replace $s\varepsilon(s)^2 \to_{s\to\infty} \infty$ by $s\varepsilon(s)^2/\log s \to_{s\to\infty} \infty$.
(41) Page 123, (3.6.5), replace $\Delta_\ell(x,y)$ by Δ_ℓ.
(42) (*) Page 124, (3.6.8), replace $R(z,y)$ by $R(x,z)$, and $S(x,z)$ by $S(z,y)$. In the second line of (3.6.9), replace $R(z,z')$ by $R(x,z)$ and $S(x,z)$ by $S(z,z')$.
(43) Page 126, line 15, replace "farther"' by "further". Same in Page 128, line -12.
(44) Page 133, line -10, replace n by N.
(45) Page 137, first display, replace $O(t^{4/3})$ by $O(t^{2/3})$, replace $O(t^{1/3})$ by $O(t^{-1/3})$.
(46) Page 142, Exercise 3.7.11, in the contour C, replace $-e^{-2\pi i/3}\infty$ by $e^{-2\pi i/3}\infty$ and replace $e^{-2\pi i/3}\infty$ by $e^{2\pi i/3}\infty$. In (3.7.31), erase the first - sign.
(47) Page 147, Equation (3.8.22), the limit in t should be as $t \to -\infty$.
(48) Page 151, line 3, add that $n' = n$ if n is even and $n' = n+1$ if n is odd.
(49) $*$ Page 151, line -6: the claim that $\Phi_A(z)$ arises from $\Psi_A(z)$ by column operations is true only when n is odd. For n even, the

conclusion that

$$\int_{A_+^r} \det \Psi_A(z) \prod_1^r dz_i = c_1 \int_{A_+^r} \det \Phi_A(z) \prod_1^r dz_i \tag{0.1}$$

is arrived at differently, as we now describe. Set

$$\Theta_A(z) = \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r}\,.$$

By applying column operations we have $\det \Psi_A(z) = \det \Theta_A(z)$. Define

$$\dot{\Theta}_A(z) =$$
$$\left[\ \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r-1} \quad \left[\begin{array}{cc} f_i(z_r) & -\epsilon(\mathbf{1}_A f)(\infty) \end{array}\right]|_{n,1}\ \right]$$

and the "deformed matrix"

$$\tilde{\Psi}_A(z) := (\Theta_A + \dot{\Theta}_A)(z)$$
$$= \left[\ \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r-1} \quad \left[\begin{array}{cc} f_i(z_r) & -\epsilon(\mathbf{1}_A f)(z_r) \end{array}\right]|_{n,1}\ \right].$$

Applying obvious column operations one gets $\det \Phi_A(z) = c_1 \det \tilde{\Psi}_A(z)$, for a nonzero constant c_1 independent of A and z. Finally, one has

$$\begin{aligned}
& \int_{A\cap[z_{r-1},\infty)} (\det \widetilde{\Psi}_A(z_1,\ldots,z_{r-1},t)dt - \det \Psi_A(z_1,\ldots,z_{r-1},t))dt \\
= & \int_{A\cap[z_{r-1},\infty)} \det \dot{\Theta}_A(z_1,\ldots,z_{r-1},t)dt \\
= & \det \left[\ \left[\begin{array}{cc} f_i(z_j) & \epsilon(\mathbf{1}_A f_i)|_{z_j}^{\infty} \end{array}\right]|_{n,r-1} \quad \left[\begin{array}{cc} \epsilon(\mathbf{1}_A f_i)|_{z_{r-1}}^{\infty} & -\epsilon(\mathbf{1}_A f)(\infty) \end{array}\right]|_{n,1}\ \right] \\
= & 0.
\end{aligned}$$

Integrating over the remaining coordinates yields (0.1).

(50) Page 190, line 18, display: replace $e^{-\beta x_i/4}$ by $e^{-\beta x_i/2}$ (recall that *standard Normal* is defined in pages 188-189).

(51) Page 208, line 4, replace $\mathrm{Mat}_{p\times q}$ by $\mathrm{Mat}_{p\times(q-p)}$.

(52) $*$ Page 208, Proposition 4.1.33: Unfortunately, Assumption (IId) in the definition of the Weyl quadruple is not satisfied in the setup described in Proposition 4.1.33. The following correction shows that the conclusion of the proposition still holds.

The only use of Assumption (IId) is in the proof of

$$\mathbb{T}_{I_n}(f_\lambda)(\mathbb{T}_{I_n}(G)) \subset \mathbb{T}_\lambda(M)\cap\mathbb{T}_\lambda(\Lambda)^\perp \tag{4.1.24}$$

in Lemma 4.1.26, see (4.1.27). Since (4.1.23) already shows that $\mathbb{T}_{I_n}(f_\lambda)(X) = [X,\lambda]$, and since the inclusion in $\mathbb{T}_\lambda(M)$ is automatic, it is enough to verify that

$$[X,\lambda]\cdot\tau = 0 \text{ for } X \in T_{I_n}(G),\ \lambda\in\Lambda \text{ and } \tau \in T_\lambda(\Lambda). \tag{0.2}$$

In particular, (0.2) should replace Assumption (IId) in the definition of the Weyl quadruple.

We now check (0.2) under the assumptions of Proposition (4.1.33). Recall that $0 < p \le q$, $n = p + q$, $0 \le r \le q - p$ and $q = p + r + s$. Fix a point

$$\lambda = \mathrm{diag}(\begin{bmatrix} x & y \\ y & I_p - x \end{bmatrix}, I_r, 0_s) \in \Lambda.$$

Recall that $x^2 + y^2 = x$. By implicit differentiation of the last matrix equation we deduce that

$$\begin{aligned} \mathbb{T}_\lambda(\Lambda) &= \{\mathrm{diag}\left(\begin{bmatrix} \xi & \eta \\ \eta & -\xi \end{bmatrix}, 0_{r+s}\right) \mid \ \xi, \eta \in \mathrm{Mat}_p \text{ are diagonal} \\ &\quad \text{and } (2x - I_p)\xi + 2y\eta = 0\}. \end{aligned}$$

Fix a tangent vector

$$\tau = \mathrm{diag}\left(\begin{bmatrix} \xi & \eta \\ \eta & -\xi \end{bmatrix}, 0_{r+s}\right) \in \mathbb{T}_\lambda(\Lambda).$$

We have

$$\mathbb{T}_{I_n}(G) = \{\mathrm{diag}(P, Q) | P \in T_{I_p}(\mathrm{U}_p(\mathbb{F})), Q \in T_{I_q}(\mathrm{U}_q(\mathbb{F}))\}.$$

We arbitrarily fix

$$\begin{array}{c} \\ p \\ p \\ r \\ s \end{array} \quad X = \begin{array}{c} \begin{array}{cccc} p & p & r & s \end{array} \\ \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix} \end{array} \in \mathbb{T}_{I_n}(G)$$

where we have broken down into blocks of the indicated sizes, and the blocks marked by $*$ will be irrelevant to the computation, and therefore we do not specify their values. We get

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix} \begin{bmatrix} x & y & 0 & 0 \\ y & I_p - x & 0 & 0 \\ 0 & 0 & I_r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} ax & ay & 0 & 0 \\ by & b(I_p - x) & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \end{bmatrix},$$

$$\begin{bmatrix} x & y & 0 & 0 \\ y & I_p - x & 0 & 0 \\ 0 & 0 & I_r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix} = \begin{bmatrix} xa & yb & * & * \\ ya & (I_p - x)b & * & * \\ 0 & * & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and therefore

$$[X, \lambda] \cdot \tau = \Re\mathrm{tr}\left(\begin{bmatrix} [a, x] & ay - yb & * & * \\ by - ya & -[b, x] & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} \xi & \eta & 0 & 0 \\ \eta & -\xi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^*\right).$$

In turn, by exploiting all the zeroes above, we get $[X,\lambda]\cdot\tau = [a,x]\cdot\xi + [b,x]\cdot\xi + (ay-yb)\cdot\eta + (by-ya)\cdot\eta$. Taking into account that x, y, ξ and η are real and diagonal, it implies that indeed $[X,\lambda]\cdot\tau = 0$ as required by (0.2).

(53) Page 214, second display, the expressions in the definitions of $D(\alpha,\beta)$ and $C(\alpha)$ should be squared.

(54) $*$ Page 219, equation (4.2.8), replace $\prod_{i=1}^k D_i \times D^r$ by

$$\prod_{i=1}^k D_i \times (D \setminus \cup_{i=1}^k D_i)^r.$$

(55) $*$ Page 244, display (4.2.51): add the assumption that μ^N is a positive measure of total mass $N!$.

(56) Page 284, 3 lines above Corollary 4.4.4, replace $a^2 m^{-1}$ by $a^{-2} m$.

(57) Page 286, Line -7, replace η_j by ζ_j.

(58) Page 303, Theorem 4.5.35, replace Edelman–Dumitriu by Dumitriu–Edelman.

(59) Page 358, (5.3.11), replace η by η' in the indices of the sum in the right hand side. (Here, η' is defined as in the proof of the lemma.)

(60) Page 361, lines -4,-5, replace $K_a(z)$ by $K_{\mu_a}(z)$.

(61) Page 429, line 4 of Lemma D.9, replace A by B.