MATH 234 - Homework 7

Prof. O'Neil

Due in recitation, April 11th, 2014

Each question is worth 10 points.

1. The Pareto distribution occurs frequently in economics. Its probability density function depends on two parameters, $c > 0$ and $\theta > 1$, and is given by the formula:

$$f(x; c, \theta) = \theta\, c^{\theta}\, x^{-\theta-1} \qquad \text{for } x \geq c.$$

   Let x_1, x_2, $\ldots$ x_n be independent observations from this distribution. Assume that c is known.

   (a) Find the maximum likelihood estimate $\hat{\theta}$ of θ.

   (b) What is the asymptotic variance (as $n \to \infty$) of $\hat{\theta}$?

   (c) Find a sufficient statistic for θ. Show that the statistic that you find is, in fact, sufficient.

2. Let x_1, $\ldots$, x_n be independent observations from a geometric distribution X, which has the discrete probability density function

$$P(X = k; p) = p\,(1-p)^{k-1}.$$

   This distribution can be used to model the number of trials necessary to obtain a success.

   (a) Find the maximum likelihood estimator $\hat{p}$ of the probability of a success in one trial, p.

   (b) What is the asymptotic variance of $\hat{p}$?

   (c) Is $T(x_1, \ldots, x_n) = \bar{x}$ a sufficient statistic? Show why or why not.

3. This question will address one way of comparing two samples. Assume that n_1 people are given treatment 1, and n_2 people are given treatment 2. Let X_1 be the number of people on treatment 1 who respond favorably to the treatment, and let X_2 be the number of people on treatment 2 who respond favorably. Assume that $X_1 \sim Binomial(n_1, p_1)$ and $X_2 \sim Binomial(n_2, p_2)$. Let $\tau = p_1 - p_2$.

   (a) Find the MLE $\hat{\tau}$ for τ.

   (b) Find the Fisher information matrix $I(p_1, p_2)$.

   (c) What is the asymptotic distribution of $\hat{\tau}$?

4. Let $x_1, \ldots, x_n \geq 0$ be I.I.D. samples from the random variable X with probability density:

$$f(x; \theta) = \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}}.$$

   Find a sufficient statistic for the parameter θ.