11/29/2022

Structured models

So far we have looked only at unstructured models which do not distinguish between age, sex, or other types. Here we will explore one type of model the <u>risk structured</u> model, which makes most sense when applied to STIs.

The most important question: how does this model change the predictions of the unstructured model?

Consider 2 classes: low risk and high risk

Assumptions:

1) Ignore demographics (birth/death)
2) Assume SIS model (treatable STIs)

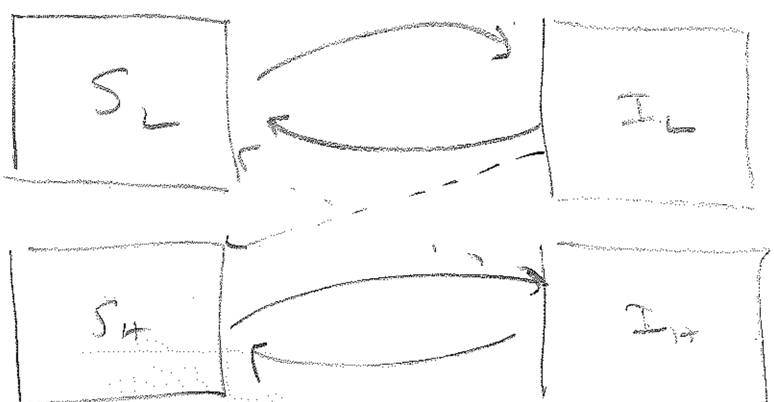
4 classes. $\hat{S}_L, \hat{S}_H, \hat{I}_L, \hat{I}_H$.

Let $N = N_L + N_H$ = total population

$S_L = \hat{S}_L/N \quad I_L = \hat{I}_L/N \quad S_H = \hat{S}_H/N \quad I_H = \hat{I}_H/N$.

which gives $S_L + I_L + S_H + I_H = 1$.

and let $n_L = S_L + I_L = N_L/N$ = fraction low risk and likewise for high risk



Let γ_H = recovery rate for high risk, γ_L = recovery rate for low risk.

β_{ij} = transmission rate from class j to class i

(# contacts per time per susceptible in class i w/ class j)

<u>ODEs</u>

$$S_L' = -\beta_{LL} S_L I_L - \beta_{LH} S_L I_H + \gamma_L I_L$$
$$I_L' = \beta_{LL} S_L I_L + \beta_{LH} S_L I_H - \gamma_L I_L$$
$$S_H' = -\beta_{HH} S_H I_H - \beta_{HL} S_H I_L + \gamma_H I_H$$
$$I_H' = \beta_{HH} S_H I_H - \beta_{HL} S_H I_L - \gamma_H I_H$$

The force of infection λ = per capita rate at which susceptibles acquire infection. Two of them in the model (low risk and high risk).

$$\lambda_L = \beta_{LL} I_L + \beta_{LH} I_H$$
$$\lambda_H = \beta_{HH} I_H + \beta_{HL} I_L.$$

Can write this in matrix form as

$$\begin{pmatrix} \lambda_L \\ \lambda_H \end{pmatrix} = \underline{\begin{pmatrix} \beta_{LL} & \beta_{LH} \\ \beta_{HL} & \beta_{HH} \end{pmatrix}} \begin{pmatrix} I_L \\ I_H \end{pmatrix}$$

This matrix has a special name - who acquires infection from whom (WAIFW). Often useful to make assumptions about this matrix to determine values of β_{ij}

1) Assortative mixing: high risk individuals are more likely to partner w/ high and low w/ low,
$$\beta_{LL}, \beta_{HH} > \beta_{LH}, \beta_{HL}.$$
transmission highest among high risk $\beta_{HH} = \max(\beta_{ij})$

2) Symmetric mixing: number of interactions between high and low risk is the same as low and high $\beta_{LH} = \beta_{HL}$

Exercise Find the DFE.

$I_L = 0 \quad I_H = 0 \qquad S_L = n_L \qquad S_H = n_H.$

Possible to show

$$R_0 = \frac{\beta_{HH} n_H + \beta_{LH} n_L}{\gamma_H} + \frac{\beta_{LL} n_L + \beta_{HL} n_H}{\gamma_L}$$

$$= R_0^H + R_0^L$$

So the total R_0 is a weighted sum of the R_0 values for each risk class. In the unstructured model, it is just β/γ. Suppose γ's were the same

$$R_0 = \frac{(\beta_{HH} + \beta_{HL}) n_H + (\beta_{LL} + \beta_{LH}) n_L}{\gamma}$$

Effective $\beta = (\beta_{HH} + \beta_{HL}) n_H + (\beta_{LL} + \beta_{LH}) n_L$.

More high risk makes β higher $= \beta_{HH} n_H + \beta_{LL} n_L + \beta_{mix}$