

## Hodgkin-Huxley Eqs for Action Potential

The model we derived previously gave us the steady state membrane potential as a function of the conductances  $g_{Na}$ ,  $g_K$ , etc.

Neurons have the ability to change the conductances, and the resulting changes in  $v$  take the shape of a pulse, "action potential."

Remember that we described the system with the eqs

$$Cv = g(V[Na^+]_i + V[K^+]_i - V[Cl^-]_i - xz) \quad (*)$$

$C$  = membrane capacitance

$$I_{Na} = -\frac{d}{dt}(gV[Na^+]_i) = g_{Na}(V - E_{Na}) + p_3$$

$$\text{where } E_{Na} = \frac{kT}{q} \log\left(\frac{[Na^+]_o}{[Na^+]_i}\right)$$

$$I_K = -\frac{d}{dt}(gV[K^+]_i) = g_K(V - E_K) - p_3 \quad E_K = \frac{kT}{q} \log\left(\frac{[K^+]_o}{[K^+]_i}\right)$$

$$I_{Cl} = -\frac{d}{dt}(-gV[Cl^-]_i) = g_{Cl}(V - E_{Cl}) \quad E_{Cl} = -\frac{kT}{q} \log\left(\frac{[Cl^-]_o}{[Cl^-]_i}\right)$$

To see how change in  $g_{Na}$ ,  $g_K$ , etc affect voltage, differentiate (\*) wrt time

$$C \frac{dv}{dt} = \frac{d}{dt}(gV[Na^+]_i) + \frac{d}{dt}(gV[K^+]_i) + \frac{d}{dt}(-gV[Cl^-]_i)$$

$$= -I_{Na} - I_K - I_{Cl}$$

So

$$Cv = -g_{Na}(V - E_{Na}) - g_K(V - E_K) - g_{Cl}(V - E_{Cl})$$

(pump currents cancel)

Technically,  $E_{Na}$  is a function of  $[Na^+]_i$ , but during an action potential these concentrations are roughly constant.

Helpful to rewrite this as

$$Cv = g(E - V)$$

where  $g = g_{Na} + g_K + g_{Cl}$  and

$$E = \frac{g_{Na}E_{Na} + g_K E_K + g_{Cl} E_{Cl}}{g_{Na} + g_K + g_{Cl}}$$

is a weighted avg of eq. potentials

\*  $V$  always approaches  $E$ . Cell adjusts  $E$  by changing  $g$ .

Since  $E_K < E_{Cl} < 0 < E_{Na}$ :

cell resting potential close to  $E_{Cl}$ .  $g_{Na} \uparrow$ ,  $v \rightarrow E_{Na}$ .

then  $g_{Na} \downarrow$ , but  $g_K \uparrow$ , so  $v \rightarrow E_K$ . then back to rest.

How do the  $\text{Na}^+$  and  $\text{K}^+$  conductances actually change?  
 Hodgkin & Huxley experiments on giant squid axon.  
 - a theory which describes their observations revolves around the  $\text{K}^+$  and  $\text{Na}^+$  channels in the cell membrane as follows:

### $\text{K}^+$ channels

- 1) Each  $\text{K}^+$  has 4 gates which can be open or closed.
- 2) all 4 gates are identical.
- 3) the gates operate independently of one another.
- 4) The probability per unit time for opening or closing a gate of a  $\text{K}^+$  channel is a specified function of voltage.

$n(t)$  = fraction of gates open at time  $t$ .

$$\frac{dn}{dt} = \alpha_n(v)(1-n) - \beta_n(v)n$$

$\alpha_n, \beta_n$  not too important, except:

$$\alpha_n'(v) > 0, \quad \beta_n'(v) < 0.$$

Increasing voltage encourages more opening and less closing. Channel is open only when all 4 gates are open.

$$g_K = \bar{g}_K n^4$$

probability channel is open.

### $\text{Na}^+$ channel

Unlike  $\text{K}^+$ , where an upward step of voltage makes  $g_K$  rise and stay there, for  $\text{Na}^+$  the upward conductance of  $\text{Na}^+$  is only maintained briefly. This means there must be channels which close in response to increasing voltage (h).

1)  $\text{Na}^+$  channel contains 3 gates which can be open or closed.

2) 3 gates are "m gates" (open when  $v$  increases)

1 gate is an "h gate" (close when  $v$  increases)

$$\frac{dm}{dt} = \alpha_m(v)(1-m) - \beta_m(v)m \quad \alpha_m'(v) > 0$$

$$\beta_m'(v) < 0$$

$$\frac{dh}{dt} = \alpha_h(v)(1-h) - \beta_h(v)h \quad \alpha_h'(v) < 0$$

$$\beta_h'(v) > 0$$

Exercise 1. Membrane conductance channel open only if 4 gates are open

$$g_{Na} = \bar{g}_{Na} m^3 h$$

A third channel - the "leakage" channel, has constant conductance

$$g_L = \bar{g}_L$$

and corresponds roughly to  $Cl^-$  channel, but isn't specific to  $Cl^-$  ions. The corresponding equilibrium potential is  $E_L$ .

Summary of equations

$$C \frac{dV}{dt} + g(V - E) = \frac{I_0(t)}{\text{applied current}}$$

$$g = g_{Na} + g_K + g_L$$

$$E = \frac{g_{Na} E_{Na} + g_K E_K + g_L E_L}{g_{Na} + g_K + g_L}$$

$$g_{Na} = \bar{g}_{Na} m^3 h$$

$$g_K = \bar{g}_K n^4$$

$$g_L = \bar{g}_L$$

the gating variables

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m$$

and likewise for  $h$  and  $n$ .