10/13/2022

Other parts of the kidney

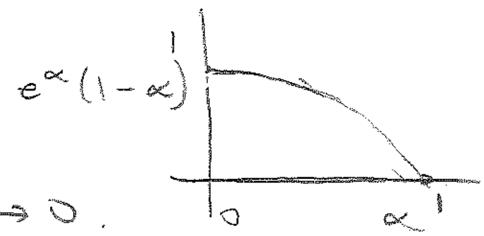
The juxtaglomerular apparatus - cells monitor tubular fluid at top of ascending limb. Monitor Na^+ concentration, constrict the blood vessels accordingly

Assume inflow $Q_1(0)$ takes on whatever value is needed to satisfy $c_2(0) = c^*$

where $c^* < c(0)$ is the target concentration

Recall $c_2(0) = c(0)\exp(\alpha)(1-\alpha)$

$$\frac{c^*}{c(0)} = \exp(\alpha)(1-\alpha).$$



This is a nonlinear equation for α

Can be solved if we assume $\frac{c^*}{c(0)} \to 0$.

Claim: $e^\alpha(1-\alpha)$ is a decreasing function for $\alpha > 0$.

Proof: $f'(\alpha) = e^\alpha(1-\alpha) - e^\alpha = -\alpha e^\alpha < 0$

Analytical solution:

if $\frac{c^*}{c(0)} := a \to 0$, then $\alpha \approx 1$

i.e., $f^*_{Na}L = Q_1(0)c(0)$.

Flux entering the loop is such that all the Na that comes in is pumped out.

Can rewrite results of previous section with c^* as a parameter (assuming a is small)

$(*)$ $Q_1(0) = f^*_{Na}L / c(0)$ $\quad [\alpha = 1]$

$-Q_2 = f^*_{Na}L / e\,c(0)$ $\quad [Q_2 = -Q_1(0)e^{-\alpha}]$

$c(L) = e\,c(0)$ $\quad [c(L) = c(0)e^{\alpha}]$

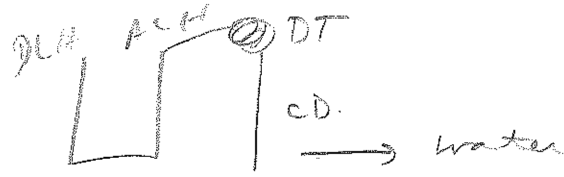
$c_2(0) = c^*$

$(*)$ - amt of inflow adjusted so that amt of Na^+ entering loop = amt pumped out of ascending loop.

<u>The distal tubule and collecting duct: concentrating + diluting modes</u>

Now comes the decision: excrete large volume of dilute urine or small volume of concentrated urine. Controlled by hormone ADH.

ADH absent = assume distal tube + CD are conduits



ADH absent - fluid leaving ascending limb becomes urine

H_2O excretion rate: $\frac{f^*_{Na} L}{e\,c(0)} = Q_2(0)$

Na^+ excretion rate: $\frac{f^*_{Na} L}{e\,c(0)} c^* = Q_2(0)\,c_2(0)$..

When ADH is present, distal tube and CD are permeable to water. As before, assume permeability is so great as to achieve equilibrium at every stage.

In distal tube - withdraw water so c = blood plasma

In collecting duct - withdraw water to equilibrate $c(x)$ at each x

$$\text{Urine conc.} = c(L) = e\,c(0)$$

Volume rate of urine production: Na^+ flux at end must be same as leaving loop

$$Q_{urine}\, e\, c(0) = Q_2(0)\, c_2(0) = \frac{f^*_{Na} L}{e\,c(0)} c^*$$

So $$Q_{urine} = \frac{f^*_{Na} L c^*}{(e\,c(0))^2}$$

and the rate of Na^+ leaving is

$$f_{Na} = Q_{urine}\, c(L) = \frac{f^*_{Na} L c^*}{e\,c(0)}$$

Conclusions: in diluting mode, urine more dilute than blood plasma, in concentrating mode more concentrated (by factor of e). Volume flow rate when ADH present = $\frac{c^*}{e\,c(0)}$ times when ADH is absent.

Recall $c^*/c(0) \ll 1$, so the flow rate is much smaller

Rate of Na^+ excretion is by necessity the same.

ADH secretes water to regulate Na^+ concentration