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Gas exchange in an alveolus (2.1–2.2)

Lungs contain 3×10^8 alveoli in which air oxygenates our blood
O_2 is picked up by blood ⟷ CO_2 enters the lungs.
Total blood flow is sum of the individual alveoli
We first will try to understand the equations governing the interaction of a gas with a liquid

Gas alone: the ideal gas law (Thomas Piche: "I don't believe in the ideal gas law")

$$PV = nkT$$

(Pressure)(Volume) = (# of molecules)(Boltzmann's constant)(absolute temp)

Also written $PV = \left(\frac{n}{N_A}\right)(kN_A)T$
$= m\,R\,T$ m = # of moles.

Pressure = force per area exerted by molecules colliding with walls of container

Suppose we have a mixture of gases, where the partial pressure of each gas is P_j and the number of molecules is n_j

$$P_j V = n_j kT$$

Sum over j, $V\sum P_j = kT\sum n_j \longrightarrow PV = nkT \longrightarrow \boxed{\frac{P_j}{P} = \frac{n_j}{n}}$ ☆

Let $c_j = n_j/V$ then $P_j = c_j kT$ (concentration in gas)

Vapor liquid equilibrium

P,T → When rate of molecules entering liquid = rate entering gas, we have equilibrium (where P, and T are the same!)

In simple solutions

At equilibrium, concentration of gas dissolved in the liquid is proportional to its partial pressure in the gas.

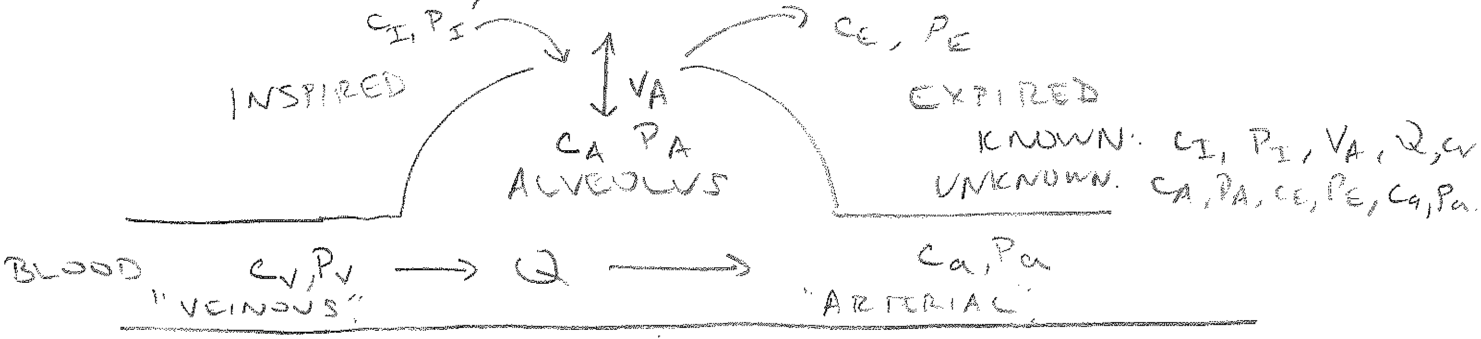
Simple solutions obey $\boxed{c_j = \sigma_j P_j}$

σ_j = "solubility" of gas $P_j = c_j/\sigma_j$ = partial pressure needed to maintain gaseous phase.

Later: we will consider $c_j = C_j(P_j)$ where C_j is a nonlinear function. But for now we will restrict ourselves to simple solutions.

EXERCISE on back

Gas transport in one alveolus (2.2)



V_A = volume of fresh air delivered to alveolus per unit time

"Veinous" and "arterial" blood refer to its chemical composition

arterial blood is O_2 rich, venous O_2 depleted

Derive eqns by making the following assumptions

1) Steady state: # of gas molecules entering per time = # leaving

ENTERING: $c_v Q + V_A c_I$ $\Longrightarrow$ $c_v Q + V_A c_I = c_a Q + V_A c_E$

LEAVING: $c_a Q + V_A c_E$

2) Expired air is alveolar air. $c_E = c_A$, $P_E = P_A$.

3) Ideal gas law: $P_A = kT c_A$ for alveolar air

4) Gas forms simple solution in blood (wrong): $\sigma P_a = c_a$

5) Equilibrium: the partial pressure of the gas = the partial pressure in the liquid $P_A = P_a$.

Next time we will solve these eqns to obtain c_A and c_a.

Combine 1) and 2)

$$Q(c_a - c_v) = V_A(c_I - c_A).$$ Unknowns c_A and c_a.

3) - 5): $c_a = \sigma P_a = \sigma P_A = \sigma k T c_A$

Solve it yourself!

$$Q(\sigma k T c_A - c_v) = V_A(c_I - c_A).$$

$$c_A(Q\sigma k T + V_A) = V_A c_I + Q c_v.$$

$$\boxed{c_A = \frac{V_A c_I + Q c_v}{Q\sigma k T + V_A}}$$

Let $r = V_A/Q$, then $$\boxed{c_A = \frac{r c_I + c_v}{r + \sigma k T}}$$

and $c_a = \sigma k T c_A$.