MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 8
Due: Tuesday 11/08 3:30 PM

1) Consider a membrane immersed in a calcium solution with external concentration $\left[Ca^{2+}\right]_o$. In this homework, the only ions inside and outside the membrane are Ca^{2+}.

(a) Write an expression for the equilibrium potential E_{Ca} of Ca^{2+}. [1 pt]

(b) Letting C be the membrane capacitance, write the ODE describing the evolution of the cell potential v as a function of E_{Ca} and g_{Ca}. [1 pt]

We will now take a closer look at the behavior of the calcium channels. Suppose that each channel contains two gates, and that the channel is open if both gates are open. Let a be the probability the first gate is open and b be the probability the second gate is open. Unlike in class, however, the second gate is open only when the first gate is open (it is not independent of the first).

(c) Let $\bar{g}_{Ca}$ be the membrane conductance when all channels are open. Write the algebraic equation for the conductance g_{Ca} as a function of a, b, and $\bar{g}_{Ca}$. [2 pts]

(d) Write an ODE for da/dt, introducing functions $\alpha_a(v)$ and $\beta_a(v)$ that describe the rate of gate opening and closing (respectively) as a function of the voltage v. [1 pt]

(e) When the voltage increases, the rate at which an open gate closes increases, and the rate at which a closed gate opens also increases. What does this mean about the sign of $\alpha_a'(v)$ and $\beta_a'(v)$? [2 pts]