MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 10
Due: Tuesday 12/06 3:30 PM

1) Let's consider a disease like the flu or covid where there are essentially two populations – healthy people (H) and vulnerable people (V). The key difference is that healthy people cannot die from the disease while vulnerable people can. With this in mind, separate the population into two compartments (H and V) with the rate of contact between classes $\beta_{HH}, \beta_{HV}, \beta_{VH}$, and β_{VV}, respectively. Let the recovery rate in class H be γ_H and in class V let it be γ_V, and denote the probability that vulnerable people die from the disease by d. You can ignore other demographics (birth and death) in this problem.

(a) Separate the healthy people into three compartments (SIR) and vulnerable people into four compartments (SIRD). Write the ODEs that govern the evolution of each compartment. [7 pts]

(b) In recitation, you'll learn how to program this kind of model in Matlab. Use the following parameters: 10% of the population is vulnerable, $d = 0.2$, $\beta_{VV} = 0$, $\beta_{HV} = 1$, and $\gamma_V = 1/20$. Then, keeping a constant $\beta_{HH}/\gamma_H = 5$, compare the pairs $(\beta_{HH}, \gamma_H) = (1/10, 1/50)\,, (1, 1/5)\,,$ and $(10, 2)$ and report what happens to the number of people who have died over the course of the epidemic. The idea here is to keep the rough measure of $R_0 = \beta_{HH}/\gamma_H$ constant among the healthy people, but vary the speed at which the epidemic spreads among healthy people, and look at how it affects the vulnerable people. [3 pts]

2) Consider a population whose growth is governed by the ODE

$$\frac{dN}{dt} = NF(N),$$

where we have the data below for $F(N)$.

N	0.5	1.5	2.5	4.2
$F(N)$	2.625	-0.625	1.125	-1.408

(a) Assuming that $F(N)$ is a continuous function, how many equilibrium points is the population guaranteed to have? [2 pts]

(b) Assuming that $F(N)$ takes the form $aN^3 + bN^2 + cN + d$, use the data to solve for the coefficients and thus find $F(N)$. [3 pts]

(c) Factor $F(N)$ to find the equilibrium points. [2 pt]

(d) Which equilibrium points are stable? Explain the meaning of this in terms of the population. When will the population size be stable? [2 pts]

(e) The table below gives several possible populations at time zero. Fill in the last row which gives the population after a long time. [3 pts]

$N(t = 0)$	0.01	1.2	1.8	2.2	3.5	5
$N(t \to \infty)$						