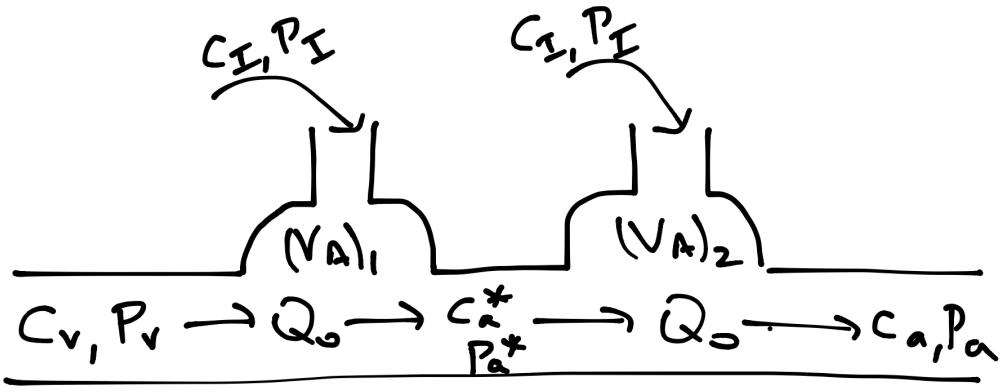
MATH/BIOL 255: Mathematics in Medicine and Biology
Homework 1
Due: Tuesday 09/13 3:30 PM

1. In class, we analyzed the case of alveoli in parallel. Consider two alveoli with equal flows of blood, $Q_1 = Q_2$. Define the ventilation to perfusion ratio for the parallel alveoli as $r_1^{(p)} = (V_A)_1/Q_1$ and $r_2^{(p)} = (V_A)_2/Q_2$.

(a) Let $Q_0 = Q_1 + Q_2$. Write an expression for the efficiency of gas transport in this case as a function of $r_1^{(p)}$ and $r_2^{(p)}$. [1 pt]

(b) When is the efficiency maximal? [1 pt]

(c) Now consider the case of two alveoli *in series* as shown below.



The blood flows with volume $Q_0 = Q_1 + Q_2$ between the alveoli, which have the same ventilation volumes as the parallel alveoli in part (a). As a result, the ventilation to perfusion ratios are defined as

$$r_1^{(s)} := (V_A)_1/Q_0 = r_1^{(p)}/2 \qquad \text{and} \qquad r_2^{(s)} := (V_A)_2/Q_0 = r_2^{(p)}/2. \tag{1}$$

Using the same equations and assumptions as in class, determine c_a^* as a function of c_I and c_v. [1 pt]

(d) Determine c_a as a function of $c_I, c_v, r_1^{(s)}$, and $r_2^{(s)}$. [3 pts]

(e) Determine the total gas flux

$$f_{\text{series}} = Q_0\,(c_a - c_v)$$

for the alveoli in series. Give your answer as a function of $P_I, P_V, r_1^{(s)}$, and $r_2^{(s)}$. [3 pts]

(f) Determine the efficiency of gas transfer in the series case,

$$E_{\text{series}} = \frac{f_{\text{series}}}{\sigma\,(P_I - P_v)\,Q_0}$$

as a function of $r_1^{(s)}$, and $r_2^{(s)}$. [2 pts]

(g) What is the difference in efficiency between the parallel and series geometries? Which one is better? To do this, you will have to convert between the ventilation to perfusion ratios using equation (1). Specifically, set $r_1^{(p)} = r_2^{(p)} = r_0$, so that $r_1^{(s)} = r_2^{(s)} = r_0/2$ and plot the efficiencies E_{parallel} and E_{series} on the same plot as a function of r_0. Use $\sigma kT = 1$ and vary r_0 between 0 and 10. You should use excel, Matlab, or some other computer program to generate your plots. [4 pts]