1

Now consider advection - diffusion with finite-volume

$$u_t + a u_x = d u_{xx}$$

Dirichlet BC gives flux directly for advection, $f_{adv} = a u$

Neumann BC gives flux directly for diffusion, $f_{diff} = -d u_x$

So for those fluxes we do not need any interpolation/extrapolation or ghost cells!

2*

If we want our method to work for $d \to 0$ (pure advection or advection-dominated flows, we must treat advection separately and realize that there is no BC at outflow for advection, only for diffusion.

So treat diffusion as already explained in lecture notes

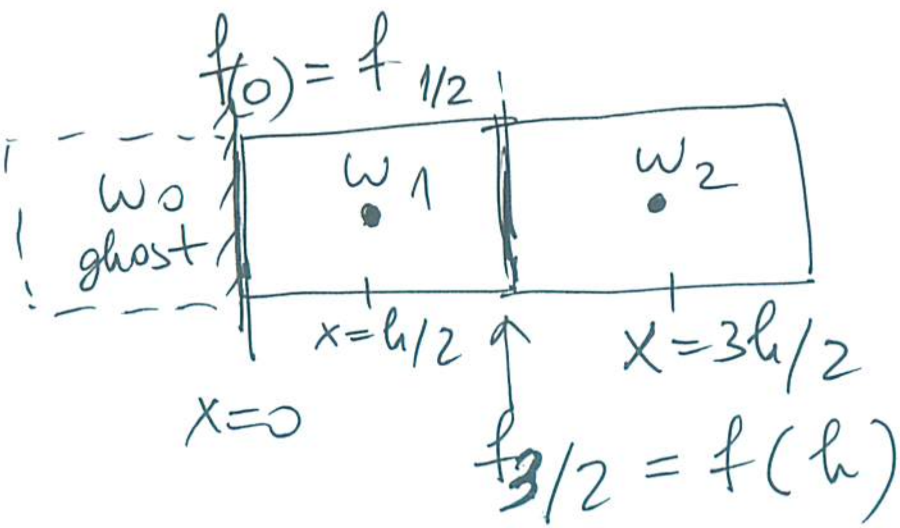
For advection, we need to consider second or third-order schemes separately.

3*

$$\begin{cases} u(0,t) = \tilde{f}(t) & \text{Dirichlet on left} \\ u_x(L_x,t) = \tilde{g}(t) & \text{for diffusion but} \end{cases}$$

does <u>not matter</u> for advection!

At the <u>right</u>, we have <u>outflow boundary</u> and at the left, <u>inflow boundary</u>



$$f_{1/2} = f(0) = a\tilde{f}(t)$$

(flux at $x = 0$)

$$f_{3/2} \approx f(x=h) = ?$$

An obvious choice is

1. Centered - advection:

$$f_{3/2} = -a\,\frac{w_1 + w_2}{2} \quad \text{(no ghost cell)}$$

this gives (this is crucial to check)

$$\frac{dw_1}{dt} = -\frac{a}{h}\left(w_1 + w_2 - w_{1/2}\right)$$

where $\quad w_{1/2} \simeq u(x=0) = \tilde{f}(t)$

A Taylor series expansion shows that this is first-order accurate at the boundary, which is ok since we get +1 order from $1/h$ in stencil.

NOTE: One can pretend here this is finite-difference

At the outflow boundary, we want a one-sided stencil (no BC) so it is most natural to use first-order upwinding at outflow 5*

$$\frac{dw_N}{dt} = -\frac{a}{h}\left(w_N - w_{N-1}\right)$$

Simple algebra shows that this is the same as using a ghost cell with linear extrapolation (MUST CHECK THIS!)

$$w_{N+1} = 2w_N - w_{N-1}$$