THEORY OF PROBABILITY: HOMEWORK 9 SOLUTIONS

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All problems below are from Ross, Chapter 4. These problems are the same in the 8th edition of the book, except for Problem 10, which is reproduced below.

Problem (2). Let $n$ be the number of distinct natural numbers that are divisors of $i$, such that $1 \leq n \leq 6$ and $1 \leq 36/n \leq 6$. Then, we get:

$$p(i) = P(X = i) = \frac{n}{36}$$

Problem (7). (just parts (a) and (d))

(a) $\{i \in \mathbb{N} : 1 \leq i \leq 6\}$

(d) $\{i \in \mathbb{N} : 5 \leq i \leq 5\}$

Problem (8). (just parts (a) and (d))

(a) Let $X$ be the maximum value to appear in two rolls, and let $n$ be the number of distinct natural numbers that are less than $i$. Then

$$p(i) = P(X = i) = \frac{n+1}{36}$$

(d) Let $X$ be the the value of the first roll minus the value of the second roll, and let $n$ be the order of $\{k \in \mathbb{N} : k \leq 6, 1 \leq i+k \leq 6\}$

$$p(i) = P(X = i) = \frac{n}{36}$$

Problem (10). Let $X$ be the winnings of a gambler. Let $p(i) = P(X = i)$ and suppose that:

$$p(0) = 1/3; p(1) = p(-1) = 13/55;$$

$$p(2) = p(-2) = 1/11; p(3) = p(-3) = 1/165;$$

Compute the conditional probability that the gambler wins $i$, $i = 1, 2, 3$, given that he wins a positive amount.

Let $F$ be the event that the gamble wins a positive amount. Then
\[ P(F) = \frac{13}{55} + \frac{1}{11} + \frac{1}{165} = \frac{1}{3} \]

\[ P(X = i|F) = \frac{p(i)}{P(F)} = 3p(i) \]

Problem (13).

\[ p(0) = (.7)(.4) \]
\[ p(500) = (.3)(.4)\frac{1}{2} + (.7)(.6)\frac{1}{2} \]
\[ p(1000) = (.3)(.4)\frac{1}{2} + (.3)\frac{1}{2}(.6)\frac{1}{2} + (.7)(.6)\frac{1}{2} \]
\[ p(1500) = 2(.3)(.6)\frac{1}{2}\frac{1}{2} \]
\[ p(2000) = (.3)(.6)\frac{1}{2}\frac{1}{2} \]

References