1. Determine the group of invertible elements \( \mathcal{O}_{K,S}^* \) of the ring \( \mathcal{O}_{K,S} \) in the following cases: a) \( K = \mathbb{Q}, S = \{2, 3\} \); b) \( K = \mathbb{Q}(\sqrt{-2}), S = \emptyset \).

2. Let \( G \) be an abelian group, \( h : G \to R \) a function that satisfies the equality

\[
h(P + Q) + h(P - Q) = 2h(P) + 2h(Q), \forall P, Q \in G.
\]

The goal is to show that \( h \) is a quadratic form: \( h(mP) = m^2h(P) \forall P \in G, m \in \mathbb{Z} \) and \( (P, Q) \mapsto \langle P, Q \rangle = h(P + Q) - h(P) - h(Q) \) is a symmetric bilinear form.

(a) Show that \( h(-P) = h(P) \) and that \( h(0) = 0 \).

(b) Show that \( h(mP) = m^2h(P) \forall P \in G, m \in \mathbb{Z} \).

(c) Show that \( \langle P + R, Q \rangle = \langle P, Q \rangle + \langle R, Q \rangle \).

(d) Deduce that \( h \) is a quadratic form.