MATH-GA 2150.001: Homework 1: some answers

1. \( J = \langle x^2 + y^2 - 1, y - 1 \rangle \subset k[x, y] \).
   
a. \( V(J) = \{(0, 1)\} \).
   
b. \( f = x \) works.
   
c. \( I(V(J)) = (x, y - 1) \).

3f. 
   
i. \( X = V(y, y^2 - xz) \subset A^2_k \); the irreducible components are \( \{y = 0, x = 0\} \) and \( \{y = 0, x = 0\} \).
   
   ii. \( X = V(x(y - x^2 + 1), y(y - x^2 + 1)) \subset A^2_k \); the irreducible components are \( \{x = 0, y = 0\} \) and \( \{y - x^2 + 1 = 0\} \).
   
   iii. \( X = V(x^2) \subset A^2_k \); \( X = \{x = 0\} \) is irreducible.

4. 
   
a. \( X = V(x^2y, (x-1)(y+1)^2) \), \( I(X) = (x(x-1), xy, (x-1)(y+1), y(y+1)) \).
   
b. \( X = V(y^2 + x^2y - x^2) \), \( I(X) = (y^2 + x^2y - x^2) \).
   
c. \( X = V(z - xy, y^2 + xz - x^2) \), \( I(X) = (z - xy, y^2 + xz - x^2) \).