Problem 1. Which of the following form vector spaces? If they do what is the dimension of the vector space? In all cases we deal with the usual addition of matrices and multiplication of a matrix by a scalar, the field $F$ being indicated.

1. Orthogonal $3 \times 3$ matrices, $F=$real numbers.
2. $3 \times 3$ diagonalizable matrices with all eigenvalues equal, $F=$ complex numbers.
3. $3 \times 3$ diagonalizable matrices with all eigenvalues distinct, $F=$ complex numbers.
4. $3 \times 3$ matrices whose eigenvectors are $(1, 0, 0), (0, 1, 0), (1, 1, 1)$, $F=$ real numbers.
5. Symmetric $5 \times 5$ matrices with trace zero, $F=$ real numbers.
6. $5 \times 5$ matrices which commute with a given matrix $B$, $F=$ real numbers.

Problem 2. Missing!

Problem 3. Let $B$ be a real $3 \times 3$ matrix with characteristic polynomial $t^3 + at + b$. Determine the characteristic polynomial of $B^2$ in terms of $a, b$.

Problem 4. Let $A$ be the complex matrix representing a transformation of the vector space $\mathbb{C}^2$ of 2-tuples over the complex numbers into itself, relative to the natural ordered basis $(1, 0), (0, 1)$. Let $A_R$ be a real $4 \times 4$ matrix representing the same transformation on the vector space of 2-tuples of complex numbers over field of the real numbers.

1. What is $A_R$ relative to the ordered basis $(1, 0), (0, 1), (i, 0), (0, i)$ in terms of the entries of $A$?
2. If $A$ is diagonal, what is $A_R$ relative to the ordered basis $(1, 0), (i, 0), (0, 1), (0, i)$ in terms of the entries of $A$?
3. How can the trace and determinant of $A_R$ be computed from the trace and determinant of $A$?
4. If $A$ and $B$ are similar $2 \times 2$ complex matrices, $A_R$ and $B_R$ defined as above relative to the same basis, then are $A_R$ and $B_R$ necessarily similar real matrices? If $A_R$ and $B_R$ are similar $4 \times 4$ real matrices, are $A$ and $B$ then necessarily similar complex matrices?

Problem 5. __

1. What is the trace of $B = I + A + A^2 + \ldots + A^{28}$ if $A = \begin{pmatrix} 4 & 3 \\ 2 & 3 \end{pmatrix}$?
2. For which $2 \times 2$ orthogonal matrices $A$ does:

$$e^A = I + \frac{A}{1!} + \frac{A^2}{2!} + \ldots$$

converge. For what $A$ does the series converge to an orthogonal matrix?

Problem 6. (Bonus Problem - Jan 2004 #4) Prove that the nonzero eigenvalues of $AB$ and $BA$ are the same, where $A$ and $B$ are rectangular matrices of sizes $n$ by $m$ and $m$ by $n$, respectively. Express the corresponding eigenvectors of $AB$ in terms of those of $BA$. 

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