Quiz #5

No credit will be given to unjustified answers. Justify all your answers completely. (Or with a proof or with a counter example) unless mentioned differently. No step should be a mystery or bring a question. The grader cannot be expected to work his way through a sprawling mess of identities presented without a coherent narrative through line. If he can’t make sense of it in finite time you could lose serious points. Coherent, readable exposition of your work is half the job in mathematics. You will lose serious points if your exposition is messy, incomplete, uses mathematical symbols not adapted...

Problems:

1. Let

\[ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 5 & 7 & 4 & 9 & 8 & 3 & 10 & 1 & 6 \end{pmatrix} \in S_{10} \]

(a) Decompose \( \sigma \) into a product of cycles and give the cycle type of \( \sigma \).

(b) Compute \( \tau \circ \sigma \circ \tau^{-1} \) for \( \tau = (1, 2) \).

(c) Are \( (1, 2, 3) \) and \( \sigma \) conjugates in \( S_{10} \)?

(d) Give the order of \( \sigma \).

(e) Give the signature of \( \sigma \).

2. Let \( H = \{ I_d, (1, 2)(3, 4), (1, 3)(2, 4), (1, 4)(2, 3) \} \).

(a) Write the group table of \( H \). Is \( H \) commutative? What are the order of the elements in \( H \)?

(b) Let \( A = \langle (1, 2)(3, 4) \rangle \) and \( B = \langle (1, 3)(2, 4) \rangle \). Prove that \( H \) is the direct product of \( A \) and \( B \).

(c) To which of the two groups of order 4 describe in class is this group isomorphic to?

(d) Is this group isomorphic to \( \mathbb{Z}/4\mathbb{Z} \)?

(e) Is \( H \subseteq A_4 \)? Give the definition of a simple group. Is \( A_4 \) a simple group?