Quiz #3

Justify all your answers completely (Or with a proof or with a counter example) unless mentioned differently.

**Problems:** Let $X = M_n(\mathbb{R})$ be the set of *all* $n \times n$ matrices with entries in $\mathbb{R}$ and let $G = GL_n(\mathbb{R}) = \{ S : \det(S) \neq 0 \}$. Elements $S \in GL_n(\mathbb{R})$ act on arbitrary matrices by similarity transformations $\tau_S(A) = SAS^{-1}$ for all $A \in M_n(\mathbb{R})$.

Here,

$$\tau : \ G \times X \rightarrow X \ 
(S,A) \rightarrow \tau_S(A) = SAS^{-1}$$

1. Prove $\tau$ defines a group action.

2. Prove that $\tau_S : X \rightarrow X$ is a bijection. (Use that the two properties that defines an action.)

3. Prove that $\Phi : G \rightarrow Per(X)$ sending $S$ to $\tau_S$ is a homomorphism of groups.

4. Define the action kernel for this group action.

5. Let $A \in M_n(\mathbb{R})$, define the Orbit of $A$ for this group action.

6. Give an explicit example such that the orbit of $A$ is not equal to $M_n(\mathbb{R})$. Is the action transitive?

7. Give a subset $Y \subseteq X$, such that there is an action of $G$ on $Y$ and this action is transitive.

8. Let $A \in M_n(\mathbb{R})$, define the stabilizer of $A$ for this group action.