Quiz #1

Justify all your answers completely (Or with a proof or with a counter example) unless mentioned differently.

Problems:

1. Prove by induction that $6$ divides $n^3 - n$ whenever $n$ is a nonnegative integer.

2. (a) Use the Euclidean algorithm to find $gcd(65, 40)$.
   
   (b) Use the Extended Euclidean algorithm to find integers $u$ and $v$ such that $65u + 40v = gcd(65, 40)$.
   
   (c) Deduce an integer $k$ such that $65k \equiv 5 \mod 40$.

3. (a) Prove that $\equiv \mod 2$ is an equivalence relation on $\mathbb{Z}$.
   
   (b) How many distinct equivalence classes are there for this equivalence relation? Describe them explicitly.
   
   (c) Describe the quotient set $\mathbb{Z}/2\mathbb{Z}$. How many element are there? What are they?