Quiz #1

Justify all your answers completely (Or with a proof or with a counter example) unless mentioned differently.

Problems:

1. (10pt) Prove by induction that for all \( n \in \mathbb{Z}_+ \),

\[
\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}
\]

2. (10 pt) Use algorithms seen in class to find two integers \( a \) and \( b \) such that \( a \cdot 6 + b \cdot 22 = \text{gcd}(6, 22) \).

3. (30pt) Denote by \( X \) the set \( \{(a, b) | a \in \mathbb{Z}, b \in \mathbb{Z}\setminus\{0\}\} \). We define a relation \( R \) on \( X \) to be:

\[(a, b) \sim_R (a', b') \iff ab' = a'b \]

(a) Prove that \( R \) is a RST relation.

(b) One wish to define the sum of two equivalence class if possible. For this, we propose to define for each \((a, b) \in X \) and each \((a', b') \in X \) to be:

i. Show that the operation \([[(a, b)]_R \oplus [(a', b')]_R := [(a, b) + (a', b')]_R = [(a + a', b + b')]_R \) is not well defined (that is not making sense).

ii. Show that \([[(a, b)]_R \oplus [(a', b')]_R := [(ab' + a'b, bb')]_R \) is well defined (that is make sense, does not depend on the choice of the representatives).

(c) (Bonus) Does the quotient \( X/R \) is for you the ”same” of some familiar set ? If yes which one. (No need to explain).