Quiz #8

Justify all your answers completely (Or with a proof or with a counter example) unless mentioned differently. No step should be a mystery or bring a question. The grader cannot be expected to work his way through a sprawling mess of identities presented without a coherent narrative through line. If he can’t make sense of it in finite time you could lose serious points. Coherent, readable exposition of your work is half the job in mathematics. You will loose serious points if your exposition is messy, incomplete, uses mathematical symbols not adapted...

BE PRECISE!!!!!!

Problems: We will describe all the group of order 10. Let $G$ be a group of order 10.

1. Describe, up to isomorphism, all the proper subgroups of $G$. Justify your answer.

2. Prove that $G$ is a semi-direct product of two of its proper subgroups (You can start by enumerating the three properties that must be satisfied in order for this to be true.)

3. Describe all the possible semi-direct products that $G$ could be, up to isomorphism in term of group of the form $\mathbb{Z}/n\mathbb{Z}$.

4. How many distinct groups of order 10 are there? Justify that there are distinct.

5. Which one is $\mathbb{Z}/10\mathbb{Z}$? $D_5$?