Quiz #2

Justify all your answers completely (Or with a proof or with a counter example) unless mentioned differently. No step should be a mystery or bring a question. The grader cannot be expected to work his way through a sprawling mess of identities presented without a coherent narrative through line. If he can’t make sense of it in finite time you could lose serious points. Coherent, readable exposition of your work is half the job in mathematics. You will loose serious points if your exposition is messy, incomplete, uses mathematical symbols not adapted...

BE PRECISE!!!!!!

Problems:

1. Determine all the groups with no proper subgroup.

2. We will redo the characterization of all groups of order 4. Let $G$ be a group of order 4. We will argue on the maximal order of the element of $G$.
   
   (a) What are the possible maximal orders? Justify.

   (b) What happens if this maximal order is 4? How is $G$? To which well know group is it isomorphic?

   (c) Give an exhaustive list of the element of $G$ and write the multiplication table for $G$, when the maximal order in not 4. Write $G$ as a direct product of subgroups, justify. To which well known group is it isomorphic?

   (d) Are those two groups constructed in (b) and (c) isomorphics? Justify.

   (e) Deduce what are the possible groups of order 4.