Problem Set #1

1 System of linear equations

The following equations are considered over the reals numbers.

Problem 1:
Find the point of intersection of the line \( x_1 + 2x_2 = -13 \) and \( 3x_1 - 2x_2 = 1 \)

Problem 2:
Solve the system:

\[
\begin{align*}
    x_1 - 5x_2 + 4x_3 &= -3 \\
    2x_1 - 7x_2 + 3x_3 &= -2 \\
    -2x_1 + x_2 + 7x_3 &= -1
\end{align*}
\]

Problem 3:
Find an equation involving \( g, h \) and \( k \) that makes this augmented matrix correspond to a consistent system:

\[
\begin{pmatrix}
1 & -4 & 7 & g \\
0 & 3 & -5 & h \\
-2 & 5 & -9 & k
\end{pmatrix}
\]

2 Row reduction and echelon forms

Problem 4:
Row reduce the matrices to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix and list the pivot column.

\[
\begin{pmatrix}
1 & 2 & 4 & 8 \\
2 & 4 & 6 & 8 \\
3 & 6 & 9 & 12
\end{pmatrix}
\]
Problem 5:
Find the general solutions of the systems whose augmented matrices are given by:
\[
\begin{pmatrix}
  1 & -2 & -1 & 4 \\
  -2 & 4 & -5 & 6
\end{pmatrix}
\]

Problem 6:
Find the interpolation polynomial \( p(t) = a_0 + a_1 t + a_2 t^2 \) for the data \( (1, 6), (2, 15), (3, 28) \).
That is, find \( a_0, a_1 \) and \( a_2 \) such that
\[
\begin{align*}
  a_0 &+ a_1 + 1^2 \times a_2 = 6 \\
  a_0 + 2 \times a_1 + 2^2 \times a_2 = 15 \\
  a_0 + 3 \times a_1 + 3^2 \times a_2 = 28
\end{align*}
\]

3 Vector equations

Problem 7:
Determine if \( b \) is a linear combination of \( a_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, a_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) and \( a_3 = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \)
and \( b = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} \).

Problem 8:
Let \( v_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, v_2 = \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} \) and \( y = \begin{pmatrix} h \\ -3 \\ -5 \end{pmatrix} \). For what values of \( h \) is \( y \) in the plane generated by \( v_1 \) and \( v_2 \)?

Problem 9:
A steam plant burns two types of coal: anthracite (A) and bituminous (B). For each ton of A burned, the plant produces 27.6 millions Btu of heat, 3100 grams (g) of sulfur dioxide, and 250 g of particulate matter (solid-particle pollutants). For each ton of B burned, the plant produces 30.2 millions Btu, 6400 g of sulfur dioxide, and 360 g of particulate matter.

1. How much heat does the steam plant produce when it burns \( x_1 \) tons of A and \( x_2 \) tons of B?
2. Suppose the output of the steam plant is described by a vector that lists the amounts of heat, sulfur dioxide, and particulate matter. Express this output as a linear combination of two vectors, assuming that the plant burns \( x_1 \) tons of A and \( x_2 \) tons of B.
3. Over a certain time period, the steam plant produced 162 million Btu of heat, 23,610 g of sulfur dioxide, and 1623 g of particulate matter. Determine how many tons of each type of coal the steam plant must have burned. Include a vector equation as part of your solution.