Quiz #4

Problems:

1. (20 pt) Let \( \phi : D_n \to \mathbb{Z}/2\mathbb{Z} \) be the map given by

\[
\phi(x) = \begin{cases} 
0 & \text{if } x \text{ is a rotation} \\
1 & \text{if } x \text{ is a reflection}
\end{cases}
\]

(a) Show \( \phi \) is a homomorphism. (Hint: Remember it is enough for this to consider the product of two reflection, the product of a reflection and a rotation, the product of two rotations.)

(b) What is \( \ker(\phi) \)? What is \( \text{Im}(\phi) \)?

Solutions:

(a) The product of two reflections is a rotation around the intersection point of the two reflection axes; the product of a reflection and a rotation is a reflection; and the product of two rotations is again a rotation.

(b) \( \ker(\phi) \simeq \mathbb{Z}/n\mathbb{Z} \) is the cyclic subgroup generated by a rotation through \( 360/n \) degrees.

\[
\text{Im}(\phi) = \mathbb{Z}/2\mathbb{Z}.
\]

2. (20pt) Consider the group \( G = S_3 \times \mathbb{Z}/6\mathbb{Z} \).

(a) Determine the set of orders of elements in \( G \), that is, the set \( \{|g| \mid g \in G\} \).

(b) Prove that \( G \) is not cyclic.

Solutions:

(a) Orders of elements in \( S_3 \): 1, 2, 3; Orders of elements in \( \mathbb{Z}/6\mathbb{Z} \): 1, 2, 3, 6; Orders of elements in \( S_3 \times \mathbb{Z}/6\mathbb{Z} \): \( \mathbb{Z} \); 1, 2, 3, 6.

(b) The order of \( G \) is 36, but there are no elements of order 36 in \( G \). Hence \( G \) is not cyclic.

3. (10 pt) List the group of order 6 without proof, up to isomorphism.

Solutions:

\( \mathbb{Z}/6\mathbb{Z}, D_3 \).