Quiz #2

Problems:

1. (30 pt) Let $F : \mathbb{Z} \to \mathbb{Z}$ be a function defined as $F(x) = 10x$.
   
   (a) Prove that $F$ is a group homomorphism.
   
   (b) Find $Ker(F)$, what can you say about $F$?
   
   (c) Find $Im(F)$.

2. (10 pt) Let $\phi : G \times X \to X$ be an action of group $G$ on the set $X = G$ given by conjugation, that $\phi(g, x) = gxg^{-1}$. Suppose $x$ is a fixed point of this action, that means that the conjugacy class of $x$ is equal to $\{x\}$ (a single point). Prove $x \in Z(G)$, i.e. $x$ is in the center.

3. (10 pt) Use the First Isomorphism Theorem to show that $(\mathbb{Z}/30\mathbb{Z})/ \langle [5] \rangle \cong \mathbb{Z}/5\mathbb{Z}$ (Note: $(\mathbb{Z}/30\mathbb{Z})/ \langle [5] \rangle$ means the group $(\mathbb{Z}/30\mathbb{Z})$ quotiented by its subgroup generated by $[5]$ the class of 5).