Quiz #2

Problems:

1. Let $G$ be the matrix group $GL(n, \mathbb{C})$ of all $n \times n$ matrices $A$ with complex entries and $\det(A) \neq 0$. This is a group under matrix multiplication, and so is the subgroup $N = SL(n, \mathbb{C})$ of matrices with determinant $+1$.
   
   (a) Prove that $SL(n, \mathbb{C})$ is a normal subgroup of $GL(n, \mathbb{C})$. What does this implies for the quotient $GL(n, \mathbb{C})/SL(n, \mathbb{C})$?
   
   (b) Let
   
   $$\tilde{\det} : \frac{GL(n, \mathbb{C})}{SL(n, \mathbb{C})} \rightarrow \mathbb{C}^\times$$
   
   Prove that $\tilde{\det}$ is a well defined isomorphism of groups. (Be careful you have to prove several properties here.)

2. (a) Give the center of $S_3$. What can you deduce about $S_3$?
   
   (b) Give the order of $(1, 3, 2)$ in $S_3$ and the group generated by $(1, 3, 2)$, to which well know group is it isomorphic to?

3. For each the following pair of groups, decide whether they are isomorphic or not. In each case, give a brief reason why.
   
   (a) $U_5$ and $U_{10}$.
   
   (b) $U_8$ and $\mathbb{Z}/4\mathbb{Z}$.
   
   (c) $U_{10}$ and $\mathbb{Z}/4\mathbb{Z}$.
   
   (d) $S_3$ and $\mathbb{Z}/6\mathbb{Z}$.

4. Give the order of $[2]$ in the group $\mathbb{Z}/6\mathbb{Z}$, give the subgroup of $\mathbb{Z}/6\mathbb{Z}$ generated by $[2]$. To which well-know group is it isomorphic to?

5. Give the order of $[5]$ in $U_6$, give the subgroup of $U_6$, generated by $[3]$. To which well-know group is it isomorphic to?