1. a) The key idea is that the limits have to be equal for both segments as $x \to a$. That means you have to solve $2a + 3 = a^2$ equation. The values of $a$ (the intersection points) are: $a = -1$ and $a = 3$. These can be found by graphically seeing the intersection point.

b) The function is not defined at number $x = \frac{\pi}{23}$ although it has a limit at this point. We can remove discontinuity by defining a function

$$f(x) = \begin{cases} \frac{\sin(x - \frac{\pi}{23})}{x - \frac{\pi}{23}} & \text{if } x \text{ is not equal } \frac{\pi}{23} \\ 1 & \text{if } x \text{ is equal } \frac{\pi}{23} \end{cases}$$

2. Section 1.5 problem 28

28. Because $x$ is continuous on $\mathbb{R}$, $\sin x$ is continuous on $\mathbb{R}$, and $x + \sin x$ is continuous on $\mathbb{R}$, the composite function

$$f(x) = \sin(x + \sin x)$$

is continuous on $\mathbb{R}$, so $\lim_{x \to \pi} f(x) = f(\pi) = \sin(\pi + \sin \pi) = \sin \pi = 0$.

3. Section 1.5, Problem 34
4. Section 1.5 Problem 38

38. Suppose that \( f(3) < 6 \). By the Intermediate Value Theorem applied to the continuous function \( f \) on the closed interval \([2, 3]\), the fact that \( f(2) = 8 > 6 \) and \( f(3) < 6 \) implies that there is a number \( c \) in \((2, 3)\) such that \( f(c) = 6 \). This contradicts the fact that the only solutions of the equation \( f(x) = 6 \) are \( x = 1 \) and \( x = 4 \). Hence, our supposition that \( f(3) < 6 \) was incorrect. It follows that \( f(3) \geq 6 \). But \( f(3) \neq 6 \) because the only solutions of \( f(x) = 6 \) are \( x = 1 \) and \( x = 4 \). Therefore, \( f(3) > 6 \).

5. Section 1.6 #40

40. Since the function has vertical asymptotes \( x = 1 \) and \( x = 3 \), the denominator of the rational function we are looking for must have factors \((x - 1)\) and \((x - 3)\). Because the horizontal asymptote is \( y = 1 \), the degree of the numerator must equal the degree of the denominator, and the ratio of the leading coefficients must be 1. One possibility is \( f(x) = \frac{x^2}{(x - 1)(x - 3)} \).

6. Section 1.6 #42.

42. (a) If \( t = \frac{1}{x} \), then \( \lim_{x \to \infty} x \sin \frac{1}{x} = \lim_{t \to 0^+} t \sin t = \lim_{t \to 0^+} \frac{\sin t}{t} = 1 \).

(b) If \( t = \frac{1}{x} \), then \( \lim_{x \to \infty} \sqrt{x} \sin \frac{1}{x} = \lim_{t \to 0^+} \sqrt{t} \sin t = \lim_{t \to 0^+} \frac{t \sin t}{\sqrt{t}} = \lim_{t \to 0^+} \sqrt{\frac{t}{t}} \frac{\sin t}{t} = 0 \cdot 1 = 0 \).

7. Section 1.6 #46.
8. Assuming \( x > 1 \)

\[
\lim_{x \to \infty} \frac{(x - \sqrt{x^2 - 1})^n + (x + \sqrt{x^2 - 1})^n}{x^n} = \lim_{x \to \infty} \frac{x(1 - \sqrt{1 - \frac{1}{x^2}})^n + x(1 + \sqrt{1 - \frac{1}{x^2}})^n}{x^n} = \lim_{x \to \infty} x^n \left[ \left(1 - \sqrt{1 - \frac{1}{x^2}}\right)^n + \left(1 + \sqrt{1 - \frac{1}{x^2}}\right)^n \right] = \left(1 - \sqrt{1 - \lim_{x \to \infty} \frac{1}{x^2}}\right)^n + \left(1 + \sqrt{1 - \lim_{x \to \infty} \frac{1}{x^2}}\right)^n = 2^n
\]

9. Optional problem
   adding and substructing 1 to numerator
\[
\lim_{x \to 0} \frac{\sqrt{\cos(x)} - \sqrt[3]{\cos(x)}}{\sin^2(x)} =
\]
\[
\lim_{x \to 0} \left( \frac{\sqrt{\cos(x)} - 1}{\sin^2(x)} + \frac{1 - \sqrt[3]{\cos(x)}}{\sin^2(x)} \right) =
\]
\[
\lim_{x \to 0} \left( \frac{\cos x - 1}{\sin^2(x)(\sqrt{\cos(x)} + 1)} + \frac{1 - \cos(x)}{\sin^2(x)(1 + \sqrt[3]{\cos x} + \sqrt[3]{\cos^2 x})} \right) =
\]
\[
\lim_{x \to 0} \frac{1 - \cos x}{x^2} \left( \frac{x}{\sin x} \right)^2 \left( -\frac{1}{\sqrt{\cos(x)} + 1} + \frac{1}{1 + \sqrt[3]{\cos x} + \sqrt{\cos^2 x}} \right) =
\]
\[
= \frac{1}{2} \left( -\frac{1}{2} + \frac{1}{3} \right) = -\frac{1}{12}
\]

(1)