1. Show that the rings of integers of \( \mathbb{Q}[\sqrt{-7}] \) and \( \mathbb{Q}[\sqrt{-11}] \) are Euclidean rings with respect to the norm \( N(a + b\sqrt{-d}) = a^2 + b^2d \).

2. Decompose \( 33 + 11\sqrt{-7} \) into irreducible elements in the ring of integers of \( \mathbb{Q}[\sqrt{-7}] \).

3. Show that \( \mathbb{Z}[\sqrt{3}, \sqrt{7}] \) is not the ring of integers of \( \mathbb{Q}[\sqrt{3}, \sqrt{7}] \).

4. Let \( A \) be a Dedekind domain. For any nonzero prime ideal \( \mathfrak{p} \) of \( A \), the ring \( A_{\mathfrak{p}} \) is a DVR and each of its nonzero ideals is a unique power of the maximal ideal \( \mathfrak{p}A_{\mathfrak{p}} \). Given a nonzero ideal \( \mathfrak{a} \) of \( A \), its valuation at \( \mathfrak{p} \) is the non-negative integer \( v_\mathfrak{p}(\mathfrak{a}) \) such that \( \mathfrak{a}A_{\mathfrak{p}} = (\mathfrak{p}A_{\mathfrak{p}})^{v_\mathfrak{p}(\mathfrak{a})} \). Show that \( v_\mathfrak{p}(\mathfrak{a}) = 0 \) for all but finitely many \( \mathfrak{p} \), and that

\[
\mathfrak{a} = \prod_\mathfrak{p} \mathfrak{p}^{v_\mathfrak{p}(\mathfrak{a})}
\]

is the decomposition of \( \mathfrak{a} \) into prime ideals (in other words, we can factorize ideals locally and then patch together the factorizations).

5. Show that every ideal of a Dedekind domain is generated by at most two elements.

6. Let \( A \) be an integral domain with fraction field \( K \), let \( \mathfrak{m} \) be a maximal ideal, and let \( \mathfrak{a} \subseteq K \) be a finitely generated fractional ideal. Show that \( (\mathfrak{a}^{-1})_{\mathfrak{m}} = (\mathfrak{a}_{\mathfrak{m}})^{-1} \).

7. Let \( \mathfrak{p}_1, \ldots, \mathfrak{p}_k \) be distinct prime ideals in a Dedekind ring. Show that

\[
\mathfrak{p}_1 \cap \cdots \cap \mathfrak{p}_k = \mathfrak{p}_1 \cdots \mathfrak{p}_k.
\]

8. Let \( A \) be a Dedekind domain with fraction field \( K \), let \( \overline{K} \) be an algebraic closure of \( K \), and let \( L, L' \subset \overline{K} \) be finite separable extensions of \( K \). Show that if a prime ideal \( \mathfrak{p} \) of \( A \) is split in \( L \) and \( L' \), then it is split in their compositum.