1. Let $A$ be a ring, let $p$ be a prime ideal, and let $a_1, \ldots, a_n$ be ideals such that $a_1 \cdots a_n \subset p$. Show that $a_i \subset p$ for some $i$.

2. Let $A$ be a ring, and let $S$ be a multiplicative subset.
   (a) Let $I \subset S^{-1}A$ be an ideal. Show that there exists an ideal $a \subset A$ such that $I = S^{-1}a = \{a/s | a \in a, s \in S\}$.
   (b) Show that the prime ideals of $S^{-1}A$ are in one-to-one correspondence with the prime ideals of $A$ that do not intersect $S$.
   (c) Suppose that $A$ is an integral domain. Show that the map $i : A \rightarrow S^{-1}A$ is injective.
   (d) Suppose that $A$ is an integral domain, and suppose that the map $i : A \rightarrow S^{-1}A$ is not surjective. Show that the map

$$\text{Spec } S^{-1}A = \{\text{prime ideals of } S^{-1}A\} \rightarrow \text{Spec } A = \{\text{prime ideals of } A\}$$

given by $I \mapsto I \cap A$ is not surjective.

3. Let $A$ be a ring, and let $f : M \rightarrow N$ be a homomorphism of $A$-modules. Show that $f$ is injective if and only if the homomorphism of $A_m$-modules $f_m : M_m \rightarrow N_m$ is injective for every maximal ideal $m$ of $A$, and show that $f$ is surjective if and only if the homomorphism of $A_m$-modules $f_m : M_m \rightarrow N_m$ is surjective for every maximal ideal $m$ of $A$.

4. Prove that

$$18 = 2 \cdot 3 \cdot 3 = (1 + \sqrt{-17})(1 - \sqrt{-17})$$

is an example of non-unique factorization into irreducibles in the ring $\mathbb{Z}[(\sqrt{-17})]$ (which is integrally closed). Find the factorizations of the ideals $(2)$, $(3)$, $(1 + \sqrt{-17})$ and $(1 - \sqrt{-17})$ into prime ideals. Hint: these prime ideals will each require two generators. Make sure to prove that they are prime!

5. A Dedekind domain with a finite number of prime ideal is a principal ideal domain. Hint: given an ideal $a$ with unique factorization $a = p_1^{n_1} \cdots p_k^{n_k}$, choose $x_i \in p_i \backslash p_i^2$ and apply the Chinese remainder theorem (1.30 in my notes) to the cosets $x_i^{n_i}$ mod $p_i^{n_i+1}$.