1. Let $K$ be a complete field with respect to a non-Archimedean absolute value. Show that a series $\sum a_n$ converges if and only if $a_n$ converges to zero.

2. Show that a $p$-adic number $a = \sum_{k=n}^{\infty} a_k p^k$ is a rational number if and only if the sequence of digits is periodic, with a finite string before the first period.

3. Write the numbers $\frac{2}{3}$ and $-\frac{2}{3}$ as 5-adic numbers.

4. Show that the field $\mathbb{Q}_p$ has no automorphisms except the identity.

5. Show that the fields $\mathbb{Q}_p$ and $\mathbb{Q}_q$ are not isomorphic unless $p = q$.

6. For which $a \in \mathbb{Z}$ is the equation $7x^2 = a$ solvable in $\mathbb{Z}_7$? For which $a \in \mathbb{Q}$ is it solvable in $\mathbb{Q}_7$?