Homework: Section 13.7

Give complete, well-written solutions to the following exercises.

1. Integrate \( g(x, y, z) = x\sqrt{y^2 + 4} \) over the surface \( S \) that is the portion of the surface \( y^2 + 4z = 16 \) that lies between the planes \( x = 0, \ x = 1, \) and \( z = 0. \)

2. Evaluate the surface integral \( \iint_S \mathbf{F} \cdot d\mathbf{S} \) for the vector field \( \mathbf{F}(x, y, z) = zi + yj + xk, \) where \( S \) is the helicoid \( \mathbf{r}(u, v) = \langle u \cos(v), u \sin(v), v \rangle, \) \( 0 \leq u \leq 1, 0 \leq v \leq \pi, \) with upward orientation.

3. Find the outward flux of the field \( \mathbf{F} = xzi + yzj + k \) across the surface of the portion of the sphere \( x^2 + y^2 + z^2 \leq 25 \) above the plane \( z = 3. \)

4. Use the surface integral in Stokes’ Theorem to calculate \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y, z) = x^2y^3i + j + zk \) and \( C \) is the intersection of the cylinder \( x^2 + y^2 = 4 \) and the hemisphere \( x^2 + y^2 + z^2 = 16, z \geq 0, \) counterclockwise when viewed from above.

5. Let \( \mathbf{n} \) be the outer unit normal of the surface \( S \) given by \( 4x^2 + 9y^2 + 36z^2 = 26, z \geq 0, \) and let \( \mathbf{F}(x, y, z) = yi + x^2j + (x^2 + y^4)^{3/2} \sin(e^{\sqrt{xyz}})k. \)

Find the value of \( \iint_S \text{curl} \ \mathbf{F} \cdot d\mathbf{S}. \)

Hint: One parametrization of the ellipse at the base of the shell is of the form \( x = a \cos(t), y = b \cos(t), \) for some constants \( a, b. \)