Homework: Section 11.1

Give complete, well-written solutions to the following exercises.

1. Study the existence of a limit at \((0,0)\) of the following functions defined on \(\mathbb{R}^2 \setminus \{(0,0)\}\)

   (a) \[ f : (x, y) \mapsto \frac{x^4y^5}{x^2 + y^2} \]

   (b) \[ f : (x, y) \mapsto \frac{x^3 - y^5}{x^2 + y^2} \]

   (c) \[ f : (x, y) \mapsto \frac{x - y}{x^2 + y^2} \]

   (d) \[ f : (x, y) \mapsto \frac{x^2y + x^2 + y^2}{xy^2} \]

2. Is the function defined by \(f(x, y) = \sin\left(\frac{x^4 + y^5}{x^2 + y^2}\right)\) if \((x, y) \neq (0, 0)\) and \(f(0,0) = 1\) continuous at \((0,0)\)?

3. Compute the partial derivatives of the following functions

   (a) \( f : (x, y) \mapsto \cos^2(x) + \sin^2(y) \)

   (b) \( f : (x, y) \mapsto \cos^2(x + \sin^2(y)) \)

   (c) \( f : (x, y) \mapsto \cos^{1+x^2}(y) \).

4. Compute the second derivatives of the function \(f : (x, y) \mapsto \exp(2x) \cos(3y)\).