Homework: Sections 13.1-13.2

Give complete, well-written solutions to the following exercises.

1. Figures (I)-(IV) contain level curves of functions $f(x, y)$. Figures (A)-(B) are their corresponding gradient fields $\nabla f(x, y)$. Match the level curves in (I)-(IV) with the gradient fields in (A)-(D). All figures have $-2 \leq x \leq 2$, $-2 \leq y \leq 2$. Provide a brief explanation.

2. Let $\mathbf{F}$ be the constant force field $\mathbf{j}$ in the figure to the right. On which of the paths $C_1, C_2, C_3$ is zero work done by $\mathbf{F}$?

3. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the oriented curve in the figure to the right, and $\mathbf{F}$ is the vector field that is constant on each of the three straight segments of $C$:

$$\mathbf{F}(x, y) = \begin{cases} \mathbf{i} & \text{on } PQ \\ 2\mathbf{i} - \mathbf{j} & \text{on } QR \\ 3\mathbf{i} + \mathbf{j} & \text{on } RS. \end{cases}$$

4. Along a curve $C$, a vector field $\mathbf{F}$ is everywhere tangent to $C$ in the direction of orientation and has constant magnitude $|\mathbf{F}| = m$. Use the definition of the line integral to explain why

$$\int_C \mathbf{F} \cdot d\mathbf{r} = m \cdot \text{Length of } C.$$
Homework: Section 13.3

Give complete, well-written solutions to the following exercises.

1. **Evaluating a work integral two ways.** Let \( \mathbf{F} = \nabla (x^3y^2) \) and let \( C \) be the path in the \( xy \)-plane from \((-1, 1)\) to \((1, 1)\) that consists of the line segment from \((-1, 1)\) to \((0, 0)\) followed by the line segment from \((0, 0)\) to \((1, 1)\). Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) in two ways:

   (a) Find parametrizations for the segments that make up \( C \), and evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \).

   (b) Use \( f(x, y) = x^3y^2 \) as a potential function for \( \mathbf{F} \) to evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \).

2. Show that the work done by a constant force field \( \mathbf{F} = ai + bj + ck \) in moving a particle along any path from \( A \) to \( B \) is

   \[ W = \mathbf{F} \cdot \overrightarrow{AB}. \]

3. Consider the vector field \( \mathbf{F} \) shown in the figure below.

   (a) Is \( \int_C \mathbf{F} \cdot d\mathbf{r} \) positive, negative, or zero?

   (b) From your answer to part (A), can you determine whether or not \( \mathbf{F} = \nabla f \) for some function \( f \)?

   (c) Which of the following formulas best fits \( \mathbf{F} \)?

   \[
   \begin{align*}
   \mathbf{F}_1 &= \frac{x}{x^2 + y^2} \mathbf{i} + \frac{y}{x^2 + y^2} \mathbf{j} \\
   \mathbf{F}_2 &= -y \mathbf{i} + x \mathbf{j} \\
   \mathbf{F}_3 &= \frac{-y}{(x^2 + y^2)^2} \mathbf{i} + \frac{x}{(x^2 + y^2)^2} \mathbf{j}.
   \end{align*}
   \]