Handout 6

6. Find the directional derivative of the function
   \[ f(x, y) = e^{2y} \ln x \]
   at the point \((1, 0, 0)\) in the direction of the vector \( \mathbf{v} = (5, -12) \).

7. Find the critical points of the function
   \[ f(x, y) = (x^2 - 4x)(2y - y^2). \]

8. Find the local maximum and minimum values and saddle point(s) of the function
   \[ f(x, y) = (x^2 - 4x)(2y - y^2). \]

9. Find the critical points of the function
   \[ f(x, y) = 2x^2 + 3y^2 - 4x - 5. \]

10. Find the extreme values of
    \[ f(x, y) = 2x^2 + 3y^2 - 4x - 5 \]
    subject to the constraint
        \[ x^2 + y^2 = 16. \]

11. Find the extreme values of
    \[ f(x, y) = 2x^2 + 3y^2 - 4x - 5 \]
    on the region described by
        \[ x^2 + y^2 \leq 16. \]
1. (10 points) Let \( f(x, y, z) = xz + e^{y-x^2} \).
   a. (4 pts) Compute the gradient \( \nabla f \).
   b. (3 pts) Find the directional derivative
      \[ D_{\langle 0, \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle} f(0, 0, 1). \]
   c. (3 pts) Find the unit vector pointing in the direction along which \( f(x, y, z) \)
      increases most rapidly at the point \( (0, 0, 1) \).

2. (10 points)
   a. (5 pts) Find an equation of the tangent plane at \( (1, 3) \) to the graph of
      \[ f(x, y) = xy^2 - xy + 3x^3y. \]
   b. (5 pts) Find an equation of the tangent plane at the point \( (0, 3, -1) \) to the
      surface
      \[ z e^x + e^{z+1} + xy + y = 3. \]

3. (10 points)
The surface contains curves \( r_1(t) = (t, t^2, t^3) \) and \( r_2(u) = (1, \sqrt{2} \sin(u), \sqrt{2} \cos(u)) \). Find the tangent plane to the surface at the point \( (1, 1, 1) \) and the line perpendicular to the tangent plane and passing through the origin \( (0, 0, 0) \).

4. (10 points) The function \( f(x, y) = x^2 + y^2 + xy + 9x \) has 1 critical point. Find it, and identify it as a local minimum, local maximum, or a saddle point.

5. (10 points) Consider the function \( f(x, y) = x^2y - 3xy^2 \). Find and classify all critical points of \( f \).

6. (10 points) Find the maximum and minimum values of \( f(x, y) = x - 2y \) subject to the constraint \( \frac{x^2}{4} + y^2 = 2 \).