Handout 1 (Coordinate systems, vectors)

**Exercise 1** Mediator plane.
Let $A(x_A, y_A, z_A)$ be a point in $\mathbb{R}^3$.
1. Let $B(x_B, y_B, z_B)$ be another point. Recall the expression for the distance $|AB|$.
2. Describe the set of points which are at equal distances from $A$ and $B$. What is it, geometrically?

**Exercise 2** Alignement problems.
1. Let $A(x_A, y_A), B(x_B, y_B), C(x_C, y_C)$ be three points in $\mathbb{R}^2$. How would you check whether they lie on a straight line or not?
2. What about three points in $\mathbb{R}^3$?
3. Application: determine whether the points lie on a straight line
   (a) $A(2, 4, 2), B(3, 7, -2), C(1, 3, 3)$
   (b) $D(0, -5, 5), E(1, -2, 4), F(3, 4, 2)$.

**Exercise 3** Analytical to geometrical description.
Describe in words the regions represented by the following equations or inequalities.
1. $x = 12, y < -4, 0 \leq z \leq 6$
2. $x^2 + y^2 + z^2 - 18z \leq 14$
3. $x^2 + y^2 + z^2 - 18x \leq 3$

**Exercise 4** General equation of a sphere.
1. Let $A(x_A, y_A, z_A)$ be a point in $\mathbb{R}^3$ and let $r > 0$. Recall the equation of the sphere of center $A$ and radius $r$.
2. Let $B(x_B, y_B, z_B)$ be another point, different from $A$. Write the equation of the sphere of center $A$ passing by $B$.

**Exercise 5** Polar and spherical coordinates.
1. How many coordinates are necessary to locate a point on a given circle in $\mathbb{R}^2$?
2. Recall what the polar coordinates in $\mathbb{R}^2$ are.
3. How many coordinates are necessary to locate a point on a given sphere in $\mathbb{R}^3$? Suggest what these can be (Hint: think of a GPS).
4. Suggest a way to locate any point in three-dimensional space (different from Cartesian coordinates!).
Exercise 6 Aztec pyramid.

1. Write the (in)-equations describing a trapezoid in $\mathbb{R}^2$.
2. Write the (in)-equations describing an Aztec pyramid in $\mathbb{R}^3$.

Exercise 7 Let $A(-1, 1)$, $B(3, 2)$, $C(4, 3)$ in $\mathbb{R}^2$.

1. Draw the equivalent representation of $\overrightarrow{AB}$ starting at the origin.
2. Compute the coordinates of the following vectors:
   
   (a) $\overrightarrow{AC}$ (draw a representation)
   
   (b) $\overrightarrow{CB} + \overrightarrow{AB}$ (draw a representation)
   
   (c) $\frac{1}{2}\overrightarrow{AB} - \frac{\overrightarrow{CB}}{|AC|}$

Exercise 8

1. Find a unit vector with the same direction as $8\vec{i} - \vec{j} + 4\vec{k}$.
2. Find a vector that has the same direction as $\langle -2, 4, 2 \rangle$ but has length 6.

Exercise 9 Suppose that $\mathbf{v}$ lies in the first quadrant and makes an angle $\pi/3$ with the positive $x$-axis, and $|\mathbf{v}| = 4$.

1. Find $\mathbf{v}$ in component form.
2. Draw $\mathbf{v}$.

Exercise 10 What is the angle between the given vector and the positive $x$-axis?

1. $\vec{i} + \sqrt{3}\vec{j}$
2. $8\vec{i} - 6\vec{j}$

Exercise 11 Someone wants to cross a canal that is 3km wide and wants to land at a point 2km upstream from the starting point. The current in the canal flows at 3.5km/h and the speed of the boat is 13km/h.

1. In what direction should one steer?
2. How long will the trip take?