STUDY GUIDE

This is a guide to some of the skills, examples, and theorems that will be on the midterm.

1. Skills

• Groups
  – Define \textit{group}
  – Define \textit{abelian}
  – Decide whether a given set and operation form a group
  – Compute using group properties
  – Construct groups of symmetries
  – Calculate with multiplication tables
  – Prove facts about groups

• Subgroups
  – Define \textit{subgroup}
  – Define \textit{cyclic group}
  – Determine whether a group is cyclic
  – Identify subgroups
  – Find subgroups of a group
  – Prove facts about subgroups

• Cosets
  – Define \textit{coset}
  – Define the \textit{index} and \textit{order} of a subgroup
  – Define the \textit{order of an element}
  – Calculate the cosets of a subgroup
  – Calculate the index of a subgroup
  – State Lagrange’s Theorem
  – Reason using Lagrange’s Theorem
  – Prove facts about cosets
  – Use counting arguments to measure the order or index of a subgroup
  – Use counting arguments to count products

• Normal subgroups
  – Define \textit{normal subgroup}
  – Identify normal subgroups
  – Find normal subgroups of a group
  – Prove facts about normal subgroups

• Quotients
  – Define \textit{quotient group}
  – Calculate $G/N$
  – Calculate with quotient groups
  – Use the Correspondence Theorem (2.7.5)
  – Prove facts about quotients

• Homomorphisms
  – Define \textit{homomorphism} and \textit{isomorphism}
  – Identify homomorphisms and isomorphisms

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– Check whether functions are well-defined
– Use properties of homomorphisms
– Calculate the image/kernel of a homomorphism
– Calculate the homomorphisms from one group to another
– Use the first isomorphism theorem to construct isomorphisms between quotients and images
– Prove facts about homomorphisms
– Determine whether two groups are isomorphic

• Automorphisms and automorphism groups
  – Define automorphism
  – Calculate $\text{Aut}(G)$
  – Prove facts about automorphisms and automorphism groups

• Products
  – Define direct product
  – Define semidirect product
  – Construct direct and semidirect products.
  – Prove facts about products

• Permutation groups
  – Define symmetric group
  – Calculate cycle decompositions of permutations
  – Calculate products of permutations
  – Determine whether a permutation is odd or even

2. Important examples

What are some subgroups, normal subgroups, quotients, etc. of these groups? What are some homomorphisms between them?

• Cyclic groups: $\mathbb{Z}_n$
• Systems of numbers: $(\mathbb{Z}, +), (\mathbb{R}, +), (\mathbb{R}^*, \times), (\mathbb{C}^*, \times), (\mathbb{Q}, +), (\mathbb{Q}^*, \times)$
• Vector spaces: $(\mathbb{R}^n, +)$
• Groups of symmetries: $\text{Sym}(\Delta)$, etc.
• Matrix groups: $\text{GL}_n(\mathbb{R}), \text{SL}_n(\mathbb{R})$
• Affine groups: $A_1$
• Direct products: $A \times B$
• Semidirect products: $A \ltimes B$
• Dihedral groups: $D_{2n} \cong \mathbb{Z}_2 \ltimes \mathbb{Z}_n$
• Permutation groups

3. Important theorems

Can you state these theorems? Can you use them to solve problems?

• Lagrange’s Theorem
• Counting products (2.5.1)
• Characterizing normality (Sec. 2.6)
• First Isomorphism Theorem (2.7.1, see also 11.3 in Judson)
• Cayley’s Theorem