Assignment 8 – Due Friday, November 7

Turn this in at the start of recitation on Friday, November 7.

1. Show that \( \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4 \) is not isomorphic to \( \mathbb{Z}_{24} \) even though 2, 3, and 4 share no common factor.

2. List all of the finite abelian groups of order 36. Show that all of them have a subgroup of order 6.

3. Suppose that \( G \) is an abelian group of order \( n \) and that \( m \) is a factor of \( n \). Show that \( G \) has a subgroup of order \( m \).

4. Give an example of a group \( G \) and normal subgroups \( H_1, \ H_2, \) and \( H_3 \) such that \( H_i \cap H_j = \{e\} \) when \( i \neq j \), but \( G \) is not isomorphic to \( H_1 \times H_2 \times H_3 \).

5. Verify Sylow’s theorem for \( S_4 \) by finding subgroups of orders 2, 4, and 8.