1. Read Section 2.5–2.6 of Herstein. (Judson covers similar material in Chapter 10.)

2. Herstein, p. 47, #5, 12 (consider the sets \( Hg_1 \cap Kg_2 \) for \( g_1, g_2 \in G \))

3. Suppose that \( H \) is a finite-index subgroup of \( G \) and \( K \) is a finite-index subgroup of \( H \) (ie., \(|G : H| < \infty \) and \(|H : K| < \infty \)). Show that \( K \) is a finite-index subgroup of \( G \) and that \(|G : K| = |G : H| \cdot |H : K|\).
   (When \( G \) has finite order, you can use Lagrange’s Theorem – what if \( G \) has infinite order?)
   (Hint: Let \( g_1, \ldots, g_m \in G \) be elements such that \( Hg_1, \ldots, Hg_m \) are the right cosets of \( H \) in \( G \), and let \( h_1, \ldots, h_n \in H \) be such that \( Kh_1, \ldots, Kh_n \) are the right cosets of \( K \) in \( H \). Show that the cosets of \( K \) in \( G \) are all of the form \( Kh_jg_i \) and that all of these sets are disjoint.)

4. Let \( A_1 \) be the group of affine functions from Assignment 1. That is, if \( m, b \in \mathbb{R} \) and \( m \neq 0 \), we define \( f_{m,b}(x) = mx + b \) and let \( A_1 = \{ f_{m,b} | m, b \in \mathbb{R}, m \neq 0 \} \). Which of the following subgroups are normal? (You may assume that each of them is a subgroup.)
   (a) \( H = \{ f_{1,b} | b \in \mathbb{R} \} \)
   (b) \( K = \{ f_{m,0} | m \in \mathbb{R}, m \neq 0 \} \)
   (c) \( L = \{ f_{m,b} | m, b \in \mathbb{R}, m > 0 \} \)

5. Herstein, p. 53, #2, 3