Intro to Math Analysis: HW8

Due: Nov 13 (Thursday)

1. Uniform continuity
Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $|f'| \leq C$. Prove that $f$ is uniformly continuous.

2. Taylor approximation
Find a polynomial which approximates $f(x) = \sqrt{x}$ within $10^{-8}$ on the interval $(4, 5)$.

3. Second derivative
Let $f : [0, 1] \to \mathbb{R}$ be a $C^2$-function with $f(0) = 0 = f(1)$ and $|f''| \leq K$. Prove that $|f'(\frac{1}{2})| \leq K/4$.

4. Upper and lower sum
Prove that $f \in R[a, b]$ if and only if for every $\varepsilon > 0$ there exist partitions $P$ and $S$ of $[a, b]$ such that $U(f, P) - L(f, S) < \varepsilon$.

5. Monotone functions are integrable
Prove that if $f$ is a monotone function on $[a, b]$, then $f \in R[a, b]$.

6. Limiting integral
Let $f$ be a positive continuous function on $[a, b]$, and let $M$ be the maximum of $f$ on $[a, b]$. Prove that

$$\lim_{n \to \infty} \left( \int_a^b f(x)^n dx \right)^{1/n} = M.$$