ARITHMETIC ALGEBRAIC GEOMETRY

August 29, 2016 - September 2, 2016

Abstracts

MONDAY

Shing-Tung Yau: *Hermitian connections on holomorphic bundles and its brief history.*

Burt Totaro: *The motive of a classifying space.*
For some finite groups G, Bogomolov and Saltman showed that the quotient of a vector space by G is not a rational variety. We show that these finite groups have other unexpected properties, for example that their Chow ring depends nontrivially on the base field. The arguments suggest some general characterizations of mixed Tate motives among all motives.

Igor Dolgachev: *Pencils of odd-dimensional quadrics in characteristic* 2.
I will describe a normal form for a smooth intersection of two odd-dimensional quadrics over an arbitrary field of characteristic 2. As one of the applications, I will show that every quartic del Pezzo surface over a perfect field of characteristic 2 has a canonical rational point and, thus, it is unirational.

Paolo Cascini: *Minimal model program in positive characteristic.*
I will survey some recent results on the minimal model program in positive characteristic, mostly focusing on the singularities that appear naturally in this program.

TUESDAY

Ivan Cheltsov: *Cylinders in del Pezzo surfaces.*
For an ample divisor H on a variety V, an H-polar cylinder in V is an open ruled affine subset whose complement is a support of an effective $\mathbb{Q}$-divisor that is $\mathbb{Q}$-rationally equivalent to H. In the case when V is a Fano variety and H is its anticanonical divisor, this notion links together affine, birational and Kahler geometries. In my talk I will show how to prove existence and non-existence of H-polar cylinders in smooth and mildly singular del Pezzo surfaces for different ample divisors H. As an application, I will answer an old question of Zaidenberg and Flenner about additive group actions on the cubic Fermat affine threefold cone. This is a

joint work with Park and Won.

Cecilia Salgado: *Classification of elliptic fibrations on certain K3 surfaces.* In joint work with A. Garbagnati, we classify all elliptic fibrations on K3 surfaces endowed with a non-symplectic involution under certain hypothesis on the involution. Let X be the K3 surface and ι the non-symplectic involution. The main idea behind this classification is to transfer the problem to classifying certain linear systems on a rational surface, namely the quotient surface X/ι.

In this talk I will focus on describing which possible linear systems arise on X/ι. Finally, I will focus on an example where X/ι is an extremal rational elliptic surface for which we obtain the classification of all elliptic fibrations on X modulo $Aut(X)$.

Olivier Benoist: *Curve classes on real threefolds.* The integral Hodge conjecture is a statement that predicts, when it holds true, what co homology classes with integral coefficients on a smooth projective complex variety are algebraic. In this talk, I will describe a variant of this statement for varieties defined over the field of real numbers. I will prove some positive results for curve classes on real threefolds, and explain applications of these results to questions specific to real algebraic geometry (algebraicity of cohomology classes of the real locus, and existence of curves of even genus). This is joint work with Olivier Wittenberg.

Brendan Hassett: *New examples of rational cubic fourfolds.*
(joint with Addington, Tschinkel, and Várilly-Alvarado)
We exhibit new examples of rational cubic fourfolds. Our examples admit fibrations in sextic del Pezzo surfaces over the projective plane, which are rational whenever they have a section. They are parametrized by a countably infinite union of codimension-two subvarieties in the moduli space.

WEDNESDAY

Eva Bayer: *Hasse principle for multinorm equations.* This is a joint work with Tingyu Lee and Parimala. A classical result of Hasse states that the norm principle holds for finite cyclic extensions of global fields, in other words local norms are global norms. We investigate the norm principle for finite dimensional commutative étale algebras over global fields; since such an algebra is a product of separable extensions, this is often called the multinorm principle. Under the assumption that the étale algebra contains a cyclic factor, we give a necessary and sufficient condition for the Hasse principle to hold.

Alexei Skorobogatov: *On arithmetic surjectivity.*
Given a morphism of varieties over a number field k, what is the probability that the induced map on k_v-points is surjective? Using log geometry we show that the set of such primes v is Frobenian in the sense of Serre and so has a density. We give a necessary and sufficient condition for this set to contain all but finitely many primes of k. This generalises a result of Denef conjectured by Colliot-Thélène. (A work in progress with Dan Loughran and Arne Smeets.)

François Charles: *Twistor spaces in positive characteristic.*
The twistor space of a hyperkahler manifold is a natural transcendental construction in complex geometry. We will discuss its analogue in arithmetic geometry, with applications to supersingular K3 surfaces and the global Torelli theorem.

THURSDAY

Tamás Szamuely: *Remarks on the étale homotopy of algebraic groups.*
We present some observations concerning low-degree étale homotopy groups, completed away from the characteristic, of (not necessarily linear) algebraic groups over an algebraically closed field. In particular, we shall explain an algebraic analogue of a classical theorem of Élie Cartan.

Anna Cadoret: *Geometric monodromy - semisimplicity and maximality.*
Let X be a connected scheme, smooth and separated over an algebraically closed field k of characteristic p, let $f : Y \to X$ be a smooth proper morphism and x a geometric point on X. We show the geometric variant of the Tate semi simplicity conjecture with $\mathbb{F}_\ell$-coefficients, namely that the action of the étale fundamental group $\pi_1(X, x)$ on the étale cohomology group $H^*(Y_x, \mathbb{F}_\ell)$ is semi simple for ℓ large enough. We also show that this is equivalent to the fact that the image of $\pi_1(X, x)$ acting on $H^*(Y_x, \mathbb{Z}_\ell)$ is as large as possible (what we call 'almost hyperspecial'). Both results are based on the fact that the tensor invariants of $\pi_1(X, x)$ acting on $H^*(Y_x, \mathbb{F}_\ell)$ are the reduction modulo-ℓ of the tensor invariants of $\pi_1(X, x)$ acting on $H^*(Y_x, \mathbb{Z}_l)$ for ℓ large enough. Another application of the geometric Tate semi simplicity conjecture with $\mathbb{F}_\ell$-coefficients is that the usual (i.e. with $\mathbb{Q}_\ell$-coefficients) arithmetic Tate conjectures (semisimplicity and fullness) imply the arithmetic Tate conjectures with $\mathbb{F}_\ell$-coefficients for $\ell \gg 0$. This is a joint work with Chun-Yin Hui and Akio Tamagawa.

Antony Várilly-Alvarado: *Abelian n-division fields of elliptic curves and Brauer groups of product Kummer & abelian surfaces.*
In this talk we will discuss uniform bounds for the size of the transcendental Brauer groups of certain one-parameter families of Kummer surfaces with fixed geometric Néron-Severi lattice. We will show, among other things, that over a number field of fixed degree and for a fixed prime p, the p-primary torsion of these Brauer groups is uniformly bounded. For n odd, we will show how to relate the existence of an n-torsion transcendental element on these Kummer surfaces to the existence of certain abelian division fields for associated non-CM elliptic curves. This is joint work with Bianca Viray.

Bjorn Poonen: *Bertini irreducibility theorems over finite fields.*
The classical Bertini irreducibility theorem states that if X is a geometrically irreducible subvariety of $\mathbb{P}^n$ over an infinite field k, and $dim X \geq 2$, then there exists a hyperplane H in $\mathbb{P}^n$ over k whose intersection with X is geometrically irreducible. This can fail if k is finite, but certain variants are true. For instance, we prove that if X is as above but k is finite, then the fraction of degree d hypersurfaces H whose intersection with X is geometrically irreducible tends to 1 as d tends to infinity. This result, which is more difficult than the Bertini smoothness theorem over finite fields proved in 2004, is joint work with François Charles.

FRIDAY

Howard Nuer: *Unirationality of moduli of special cubic fourfolds and K3 surfaces.*
We provide explicit descriptions of the generic members of Hassett's divisors C_d for relevant $18 \leq d \leq 38$ and for $d = 44$, giving unirationality of these C_d. It follows as a corollary that the moduli space N_d of polarized K3 surfaces of degree d is unirational for $d = 14, 26, 38$. The case $d = 26$ is entirely new, while the other two cases have been previously proven by Mukai. We also explain the construction of what we conjecture to be a new family of hyperkähler manifolds which are not birational to any moduli space of (twisted) sheaves on a K3 surface.

Tony Pantev: *Constructing automorphic sheaves with Hodge theory.*
I will explain a geometric construction which utilizes non-abelian Hodge theory to produce Hecke eigensheaves. I will focus on a specific example of the construction building automorphic sheaves on the moduli space of rank two bundles on the projective line with parabolic structure at five points. This is a joint work with Ron Donagi.