HW 2 Due Feb. 17

1. Consider a function $f \in L^1$ which admits a weak derivative in $L^1(\mathbb{R})$. Show that there exists a continuous function $g$ of which agrees with $f$ almost everywhere.

2. @ If $0 < s < 1$, find a function in $H^s(\mathbb{R})$ which is not bounded.
   @ If $s = 1$, find a function in $H^1(\mathbb{R})$ which is not bounded (think of a logarithmic singularity $\sim (\ln |x|)^s$).
   @ If $s < 1$, find a function in $H^s(\mathbb{R})$ which is not bounded on any open set.

3. @ If $d \geq 2$, show that the trace inequality
   $$\|f\|_{H^{\frac{d}{2}}(\mathbb{R}^{d-1})} \leq C \|f\|_{H^1(\mathbb{R}^d)}$$
   cannot be improved by replacing $\frac{1}{2}$ by $\delta > \frac{1}{2}$ on the right-hand side.
   (Give an example)

   @ We want to find a lifting operator which, to any $g \in H^s(\mathbb{R}^{d-1})$, associates $Lg \in H^{s+\frac{1}{2}}(\mathbb{R}^{d-1})$ such that $(Lg)|_{\mathbb{R}^{d-1}} = g$ (this shows that the trace operator is onto). Show that
   $$L(v)(x) = \frac{1}{(2\pi)^{d-1}/2} \int_{\mathbb{R}^{d-1}} e^{ix \cdot \xi'} \chi(x_d < \xi') \hat{v}(\xi) d\xi'$$
   has this property.