Homework 8
Due: 2:00pm April 7, 2016

Each problem is worth 10 points.

Exercise 1: Solve the following initial value problems analytically:
1. \( y' = t^3, \ y(0) = 0 \)
2. \( y' = 2y, \ y(1) = 3 \)
3. \( y' = ay + b, \ y(0) = y_0 \) (Assume that \( a \) and \( b \) are scalars. Hint: Multiply by the integrating factor \( e^{-ta} \) and integrate from 0 to \( T \))

Exercise 2: Verify that the function \( y(t) = t^{3/2} \) solve the initial value problem
\[ y' = \frac{3}{2} y^{1/3}, \quad y(0) = 0. \]
Apply Euler’s method to this problem and explain why the numerical approximation differs from the solution \( t^{3/2} \).

Exercise 3: Write down the result of applying one step of Euler’s method to the initial value problem
\[ y' = (t + 1)e^{-y}, \ y(0) = 0 \] using step size \( h = 0.1 \). Do the same for the midpoint method and for Heun’s method.

Exercise 4: Write down the result of applying one step of Euler’s method to the predator-prey equations:
\[ R'(t) = (2 - F(t)) R(t) \]
\[ F'(t) = (R(t) - 2) F(t), \]
starting with \( R(0) = 2 \) and \( F(0) = 1 \) and using step size \( h = 0.1 \). Do the same for the midpoint method and for Heun’s method.