Homework 5

Due: 2:00pm Mar. 3rd, 2016

Each problem is worth 10 points.

Exercise 1 [Muller's method]: When solving an equation of the form $f(x) = 0$, Newton's method uses explicit derivative information of f to calculate the tangent line and its x-axis intercept (its root). The secant method uses two points, $(x_{k-1}, f(x_{k-1}))$ and $(x_k, f(x_k))$, to fit a linear function approximating the tangent line and then finds its root.

Muller's method uses three points, $(x_{k-2}, f(x_{k-2}))$, $(x_{k-1}, f(x_{k-1}))$, and $(x_k, f(x_k))$, to fit a parabola. It then uses the quadratic formula to find the root of this parabola that is closest to x_k. Derive Muller's method. To obtain full credit, you must show your work.

Exercise 2 [Chebyshev polynomials]: We provided two equivalent formulas defining Chebyshev polynomials:

$$T_n(x) = \cos(n \arccos x)$$

and

$$\begin{aligned} T_0(x) &= 1, \\ T_1(x) &= x, \\ T_{n+1}(x) &= 2x\, T_n(x) - T_{n-1}(x). \end{aligned}$$

Show (prove) that these formulas are equivalent.

Hint: Use formulas for the cosine of a sum and difference of two angles to show that

$$\cos((n+1) \arccos x) = 2x \, \cos(n \arccos x) - \cos((n-1) \arccos x).$$

Exercise 3 [Runge's function]: Runge's function is given by:

$$f(x) = \frac{1}{1+x^2}.$$

in MATLAB, compute the Lagrange interpolant of f on the interval $[-5, 5]$ using the equispaced points

$$x_j = -5 + \frac{10(j-1)}{n-1}, \qquad j = 1, \ldots, n$$

for $n = 5, 10, 20, 40$. Plot each all of these functions at 200 equispaced points on $[-5, 5]$ on the same graph. Turn in this plot.

- continued -

Repeat the exercise, but now instead of using equispaced interpolating nodes, use Chebyshev nodes:

$$t_j = \cos\left(\frac{\pi(j-1)}{(n-1)}\right), \qquad j = 1, \ldots, n.$$

Plot each of these functions for $n = 5, 10, 20, 40$ and turn in the plots.