Homework 4

Due: 2:00pm Feb. 25th, 2016

Each problem is worth 10 points.

Exercise 1 [Conditioning]: Compute the condition number of the matrix A:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 9 \end{bmatrix}$$

Note: Do NOT just turn in a number that you computed in MATLAB without deriving how it was computed.

Exercise 2 [Cholesky decomposition]: The Cholesky decomposition is a special case of an LU decomposition for a symmetric positive definite (SPD) matrix. For an SPD matrix A, the Cholesky decomposition is:

$$A = LL^T,$$

where L is lower-triangular. Write a routine in MATLAB to compute the Cholesky factorization (without pivoting) of the $n \times n$ matrix A_n:

$$A_n = \begin{bmatrix} n & n-1 & n-2 & \cdots & 1 \\ n-1 & n & n-1 & \cdots & 2 \\ n-2 & n-1 & n & \cdots & 3 \\ \vdots & & & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & n \end{bmatrix}$$

The calling sequence of the function should be:

```
[lmat] = mycholesky(n)
```

Turn in your code, along with a printout of the output of the following MATLAB commands in `short e` format (see MATLAB preferences):

```
n = 6;
[lmat] = mycholesky(n)
```

You must also email me (`oneil@nyu.edu`) a copy of your code that I can execute.

- continued -

Exercise 3 [Eigenvalues and eigenvectors]: Let A be an $n \times n$ symmetric matrix with real-valued entries and n distinct non-zero eigenvalues. Prove that A has n linearly independent, *orthogonal* eigenvectors and that all the eigenvalues are real (i.e. not complex).

Exercise 4 [Norms]: Show that for all vectors $\boldsymbol{u}$ of length n:

1. $\|\boldsymbol{u}\|_\infty \leq \|\boldsymbol{u}\|_2 \leq \sqrt{n}\,\|\boldsymbol{u}\|_\infty$
2. $\|\boldsymbol{u}\|_2 \leq \|\boldsymbol{u}\|_1$
3. $\|\boldsymbol{u}\|_1 \leq n\,\|\boldsymbol{u}\|_\infty$