Homework 11

Due: 2:00pm April 28, 2016

Each problem is worth 10 points.

Exercise 1 : Let the matrix $\mathbf{A}$ have eigenvalues $\lambda_1 = -\lambda_2 > |\lambda_3| \geq \ldots \geq |\lambda_n|$. Determine a shift that can be used in the power method in order to calculate λ_1.

Exercise 2 : Let the matrix $\mathbf{A}$ have eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_n$ (i.e. all real, but not necessarily positive). What shift should be used int he power method in order to make it converge most rapidly to λ_1? And what shift should be used to make the power method converge most rapidly to λ_n?

Exercise 3 : Let the eigenvalues of $n \times n$ matrix $\mathbf{A}$ be $2, 4, 8, 16, 32, \ldots, 2^n$. If you are allowed use the inverse power method with shifts $s = 2^\ell + 1$, for some ℓ, what is the fastest rate that the scheme can converge to $\lambda_k = 2^k$?

Exercise 4 : Let $\mathbf{A}$ be an $n \times n$ real symmetric positive definitely matrix. Prove that the solution to the system $\mathbf{Ax} = \mathbf{b}$ is the unique minimizer of the function:

$$f(\mathbf{x}) = \frac{1}{2}(\mathbf{x}, \mathbf{Ax}) - (\mathbf{x}, \mathbf{b}).$$